A STUDY OF FACTORS AFFECTING THRUST AUGMENTATION

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A THESIS

by

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[1947]

Submitted in partial fulfillment of the requirements for the degree of Master of Engineering at Rensselaer Polytechnic Institute, Troy, New York.

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LIST OF CURVES

LIST OF TABLES

LIST OF SYMBOLS

A - Flow area sq. ft. (on thermodynamic equations) mixing section.

A - Flow area mixing section sq. in. (air ejector dimensions).

a - Flow area sq. in. primary jet.

F - Thrust Augmentation, lbs.

g - Acceleration of gravity, 32.2 ft. per sec.^{2}

M - Mach number

P_o - Total pressure, lbs per sq. ft. abs.

P - Static pressure lbs. per sq. ft. abs.

R - Gas constant, 53.3 for air.

T_o - Total temperature, degrees Rankine.

T - Static temperature, degrees Rankine.

V - Velocity ft. per sec.

W - Flow, lbs. per sec.

Cp - Specific heat at constant pressure, BTU per lb. per deg. R.

ρ - Mass density, slugs per cu. ft.

γ - (Constant) ratio of specific heats, 1.395 for air.

Subscripts and superscripts used will have the following meanings:

$(\ )_1$ - Primary air.

$(\ )_2$ - Secondary air at mixing section.

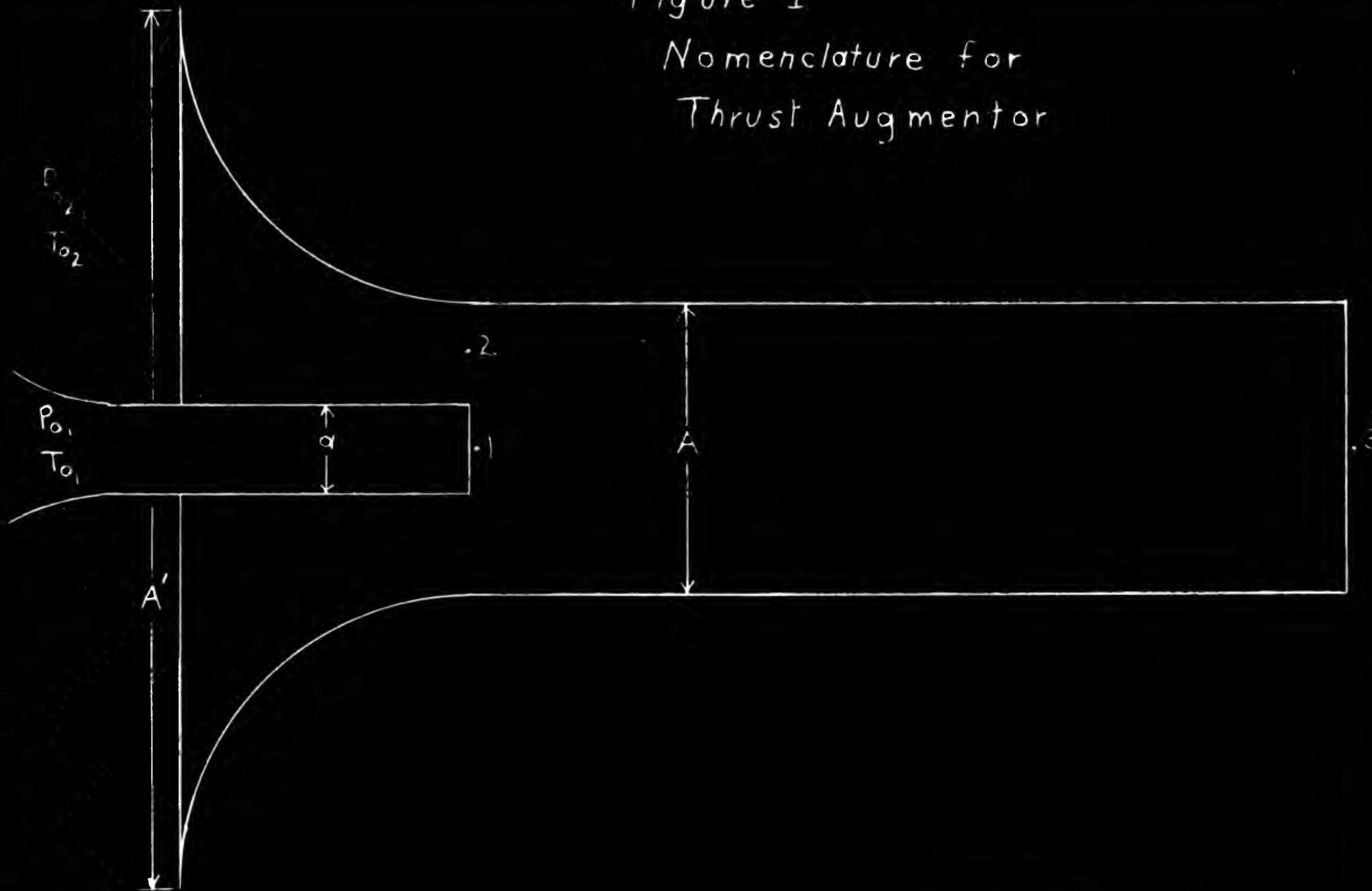
$(\ )_3$ - Mixed stream at augmentor exit.

$(\ )_2^1$ - Secondary air at augmentor entrance.

$(\ )_*$ - Bibliography reference.

Figure I

Nomenclature for Thrust Augmentor



Thrust Augmentor

INTRODUCTION

The purpose of thrust augmentation is to transfer the kinetic energy leaving a jet to a larger mass of air by providing some material boundary upon which this larger mass can react. The additional thrust force is derived by the differences in fluid pressures on the surfaces of the augmentor. If a negative static pressure exists on the inner surface of a convergent shape by virtue of an increase in velocity from a total pressure common to both surfaces, the thrust comes from the difference between the internal and external integrated pressures.

In general, a jet directed into an augmentor mixes with and accelerates a larger quantity of low velocity secondary air. The discharge from the ejector will be a large mass of air with a lower velocity than that of the primary jet, and at some static pressure higher than that at the throat of the augmentor.

PURPOSE

To determine the effect of various temperature ratios, pressure ratios, and area ratios upon the amount of static augmentation obtained with the view of finding the point of optimum design consistent with physical limitations.

HISTORY

The published material on air ejectors is voluminous but the application of air ejector theory to thrust augmentation has had very little coverage. At the present time, a large part of the thrust augmentation work that is being done is in a restricted category and the author was unsuccessful in obtaining any of the late reports.

The first tests of a thrust augmentor were made by Jacobs and Shoemaker in 1927.(1)* They found that a maximum thrust of 1.4 times the theoretical free jet reaction. Mr. Donald C. Berkey of the General Electric Company also found experimentally that thrust was increased between 40 and 50 per cent by the addition of a thrust augmentor. (2)*

In general, as stated above, the test results and theory of thrust augmentation have not been very thoroughly covered to date.

PART I

THEORY

Fundamental principles.

The action of a jet is to accelerate a mass of air rearward producing a thrust which is equal to the mass times the acceleration. The greater the velocity in the wake the greater are the losses.

If this high velocity wake can be used to transfer energy to a larger mass of air, the momentum will be increased and the thrust of a unit would be increased, provided the losses would not be excessive. In effect, then, the exhaust would be a large mass of air at a moderate velocity rather than a small mass at a high velocity.

Design Parameters.

(a) Mixing tube length--Mixing tube length is defined as the distance from the exit of the primary nozzle to the end of the straight mixing duct. In the following work it is assumed that the mixing is complete and the pressure across the entrance to the mixing tube is constant. However, some length is needed to smooth the flow. If the mixing length is increased the friction effects become predominant and performance will be decreased. In this thesis, it was assumed that the mixing tube length would be found experimentally to

bring the Mach number to such a value that the static pressure of the discharge would be equal to atmospheric pressure. Other investigators have found that an L/D of from 4 to 8 is the optimum. One investigator found that an L/D of about 7 was the best. (2)*

(b) Mixing section area ratio. This ratio is the ratio of mixing tube area to the area of the primary jet. This is one of the variables in the following analysis and will be discussed further.

(c) Ratio of mixing tube area to thrust augmentor entrance area. Since the difference between these two areas is the projected area upon which the external pressure acts, it is to be expected that augmentation will increase as the area difference increases.

(d) Temperature ratio. This is the ratio of the temperature of the primary jet to that of the secondary air. This will be covered by later analysis.

(e) Pressure ratio. The total pressure ratio of the primary stream to that of the secondary stream. This will also be covered by a later analysis.

Theoretical Analysis.

(1) Equations for calculation of constant area mixing air ejector.

Since a thrust augmentor is basically an air

ejector, the air ejector equations are applicable. It has been shown by several investigators (2)* (3)* that maximum augmentation will be obtained from a constant area mixing ejector. This is fairly obvious since for a constant pressure mixing, a diverging section would be required and the integrated forces would be decreased by the forces acting on the diverging section.

The following assumptions were made.

(1) The gases are air with constant specific heats.

(2) The ratio of specific heats is 1.395.

(3) Total momentum per second is constant.

(4) The expansion of secondary air into the mixing section is reversible.

(5) The weight of fuel added in the primary jet is negligible compared to that of the air.

The theoretical analysis of air ejectors was taken from the analysis presented by Prof. Neil P. Bailey in his Thermodynamics of High Velocity Flow. (4)*

In the constant area section of an air ejector, if wall friction is ignored, the total momentum per second at 1-2 is the same as that at 3, or

$$P_1 a + \rho_1 v_1 a v_1 + P_2(A-a) + \rho_2 v_2 (A-a) v_2 = P_3 A + \rho_3 v_3 A v_3 \quad \ldots\ldots(1)$$

but

$$\rho = \frac{P}{gRT} \quad \dots\dots (2)$$

$$P_1 a + \frac{P_1}{gRT_1} v_1^2 a + P_2(A-a) + \frac{P_2}{gRT_2} v_2^2 (A-a) = P_3 A + \frac{P_3}{gRT_3} v_3^2 A$$

$$M = \frac{v}{\sqrt{\gamma gRT}} \quad \dots\dots (3)$$

$$P_1 a + P_1 a \gamma M_1^2 + P_2(A-a) + P_2(A-a)\gamma M_2^2 = P_3 A + P_3 A \gamma M_3^2$$

$$P_1 a(1+\gamma M_1^2) + P_2(A-a)(1+\gamma M_2^2) = P_3 A(1+\gamma M_3^2) \quad \dots\dots (4)$$

but from (4)*

$$\frac{W\sqrt{T_o}}{PA} = M\sqrt{\frac{\gamma g}{R}\left[1 + \frac{\gamma-1}{2} M^2\right]} \quad \dots\dots (5)$$

$$PA = \frac{W\sqrt{T_o}}{M\sqrt{\frac{\gamma g}{R}\left[1 + \frac{\gamma-1}{2} M^2\right]}} \quad \dots\dots (6)$$

$$W_1 + W_2 = W_3 \quad \dots\dots (7)$$

(4),(6) and (7) gives

$$\frac{W_1\sqrt{T_{o_1}}(1+\gamma M_1^2)}{M_1\sqrt{\frac{\gamma g}{R}\left[1 + \frac{\gamma-1}{2} M_1^2\right]}} + \frac{W_2\sqrt{T_{o_2}}(1+\gamma M_2^2)}{M_2\sqrt{\frac{\gamma g}{R}\left[1 + \frac{\gamma-1}{2} M_2^2\right]}} = \frac{(W_1+W_2)\sqrt{T_{o_3}}(1+\gamma M_3^2)}{M_3\sqrt{\frac{\gamma g}{R}\left[1 + \frac{\gamma-1}{2} M_3^2\right]}} \quad (8)$$

A heat balance gives,

$$W_1 C_p T_{o_1} + W_2 C_p T_{o_2} = (W_1 + W_2) C_p T_{o_3} \quad \dots\dots (9)$$

Assuming Cp constant,

$$T_{o_3} = \frac{T_{o_1} + \dfrac{W_2}{W_1} T_{o_2}}{1 + \dfrac{W_2}{W_1}} \qquad \text{.................................(10)}$$

Combining (8) &(10)

$$\frac{W_1 \sqrt{T_{o_1}}\,(1 + \gamma M_1^2)}{M_1 \sqrt{\dfrac{\gamma g}{R}\left[1 + \dfrac{\gamma - 1}{2} M_1^2\right]}} + \frac{W_2 \sqrt{T_{o_2}}\,(1 + \gamma M_2^2)}{M_2 \sqrt{\dfrac{\gamma g}{R}\left[1 + \dfrac{\gamma - 1}{2} M_2^2\right]}} = \frac{(W_1 + W_2)\left[\dfrac{T_{o_1} + \dfrac{W_2}{W_1} T_{o_2}}{1 + \dfrac{W_2}{W_1}}\right]^{\frac{1}{2}} (1 + \gamma M_3^2)}{M_3 \sqrt{\dfrac{\gamma g}{R}\left[1 + \dfrac{\gamma - 1}{2} M_3^2\right]}} \qquad \text{.......(11)}$$

or

$$\frac{1 + \gamma M_1^2}{M_1 \sqrt{\dfrac{\gamma g}{R}\left[1 + \dfrac{\gamma - 1}{2} M_1^2\right]}} + \frac{W_2}{W_1}\sqrt{\frac{T_{o_2}}{T_{o_1}}}\;\frac{(1 + \gamma M_2^2)}{M_2 \sqrt{\dfrac{\gamma g}{R}\left[1 + \dfrac{\gamma - 1}{2} M_2^2\right]}} =$$

$$\sqrt{\left(1 + \frac{W_2}{W_1}\right)\left[1 + \frac{W_2}{W_1}\,\frac{T_{o_2}}{T_{o_1}}\right]}\;\frac{1 + \gamma M_3^2}{M_3 \sqrt{\dfrac{\gamma g}{R}\left[1 + \dfrac{\gamma - 1}{2} M_3^2\right]}} \qquad \text{..(12)}$$

Which gives,

$$\frac{(1+\gamma M_3^2)}{M_3\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M_3^2\right)}} = \frac{1}{\sqrt{\left(1+\frac{W_2}{W_1}\right)\left(1+\frac{W_2}{W_1}\frac{T_{o_2}}{T_{o_1}}\right)}} \times$$

$$\left[\frac{1+\gamma M_1^2}{M_1\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M_1^2\right)}} + \frac{W_2}{W_1}\sqrt{\frac{T_{o_2}}{T_{o_1}}}\;\frac{1+\gamma M_2^2}{M_2\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M_2^2\right)}}\right] \text{.....(13)}$$

The above equation can be used for solution of M_3; provided that M_1, M_2, and $\frac{T_{o_2}}{T_{o_1}}$ are known. To facilitate solution, plots of $\frac{(1+\gamma M^2)}{M\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)}}$ vs M are included in curve numbers 1-L to 1-P inclusive.

Values of the theoretical weight ratio may be found from the dimensions of the specific air ejector and a plot of the function $M\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)}$ vs M (curves 1-H to 1-K) from equation (5) as follows:

$$\frac{W_1\sqrt{T_{o_1}}}{aP_1} = M_1\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M_1^2\right)} \text{ ....(14)}$$

$$\frac{W_2\sqrt{T_{o_2}}}{(A-a)P_2} = M_2\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M_2^2\right)} \text{ ....(15)}$$

For $P_1 = P_2$,

$$\frac{W_2}{W_1} = \frac{(A-a)}{a}\sqrt{\frac{T_{o_1}}{T_{o_2}}}\;\frac{M_2\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M_2^2\right)}}{M_1\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M_1^2\right)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

With M_1 and M_2 known, then $\frac{W_2}{W_1}$ can be calculated from (16) and M_3 calculated from equation (13). $\frac{W_3\sqrt{T_{O_3}}}{AP_3}$ then can be found from curves 1-H to 1-K.

Calculation of P_3/P_1.

$$aP_1 = \frac{W_1\sqrt{T_{o_1}}}{\left(\frac{W_1\sqrt{T_{o_1}}}{aP_1}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

and,

$$(A-a)P_1 = \frac{W_2\sqrt{T_{o2}}}{\left(\frac{W_2\sqrt{T_{o2}}}{(A-a)P_1}\right)} = AP_1 - aP_1 \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

therefore,

$$AP_1 = \frac{W_2\sqrt{T_{o2}}}{\left(\frac{W_2\sqrt{T_{o2}}}{(A-a)\,P_1}\right)} + \frac{W_1\sqrt{T_{o1}}}{\left(\frac{W_1\sqrt{T_{o1}}}{aP_1}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

From equation (10),

$$T_{o3} = \frac{W_1T_{o1} + W_2T_{o2}}{W_1 + W_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

or,

$$\sqrt{T_{o_3}} = \sqrt{\frac{W_1\ T_{o_1} + W_2\ T_{o_2}}{W_1 + W_2}} \qquad \text{.........................(21)}$$

then,

$$(W_1 + W_2)\sqrt{T_{o_3}} = \sqrt{(W_1 + W_2)(W_1\ T_{o_1} + W_2\ T_{o_2})} \qquad \text{................(22)}$$

and

$$\frac{(W_1 + W_2)\sqrt{T_{o_3}}}{AP_3} = \frac{\sqrt{(W_1 + W_2)(W_1\ T_{o_1} + W_2\ T_{o_2})}}{A_1 P_3} \qquad \text{..............(23)}$$

$$AP_3 = AP_1 \frac{P_3}{P_1} \qquad \text{..(24)}$$

Combining equations (19), (23), and (24),

$$\frac{(W_1 + W_2)\sqrt{T_{o_3}}}{AP_3} = \frac{\sqrt{(W_1 + W_2)(W_1 T_{o_1} + W_2\ T_{o_2})}}{\frac{P_3}{P_1}\left[\frac{W_2\sqrt{T_{o_2}}}{\frac{W_2\sqrt{T_{o_2}}}{(A-a)P_2}} + \frac{W_1\sqrt{T_{o_1}}}{\frac{W_1\sqrt{T_{o_1}}}{\frac{W_1\sqrt{T_{o_1}}}{aP_1}}}\right]} \qquad \text{............(25)}$$

Dividing numerator and denominator by $W_1\ T_{o_1}$

$$\frac{P_3}{P_1} = \frac{\sqrt{\left(1 + \frac{W_2}{W_1}\right)\left(1 + \frac{W_2}{W_1}\frac{T_{o_2}}{T_{o_1}}\right)}}{\frac{(W_1 + W_2)\sqrt{T_{o_3}}}{AP_3}\left[\frac{\frac{W_2}{W_1}\sqrt{\frac{T_{o_2}}{T_{o_1}}}}{\frac{W_2\sqrt{T_{o_2}}}{(A-a)P_2}} + \frac{1}{\frac{W_1\sqrt{T_{o_1}}}{aP_1}}\right]} \qquad \text{.............(26)}$$

From equation (26) the static pressure at the point of mixing can be calculated.

Calculation of $M_{2'}$ and $P_o/P_{2'}$

For any value of M_2 and with given physical dimensions for the augmentor, $M_{2'}$ can be calculated.

From the relation (4)*

$$\frac{A}{A_o} = \frac{M_o}{M}\left(\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_o^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \text{.................................(27)}$$

Curve 1-F is a plot of the above relation vs M using A_o as unity where $M_o = 1.0$. This curve gives the value $A-a/A_o$ at M_2.

Then

$$\frac{(A-a)}{A_o} \times \frac{(A'-a)}{A_o} = \frac{(A'-a)}{A_o} \qquad \text{..............................(28)}$$

The value of $M_{2'}$ can be found at the value of $A'-a/A_o$ on curve 1-G.

To find $P_o/P_{2'}$ the following relation from (1) is used:

$$\frac{P_o}{P} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{...................................(29)}$$

Curve 1-A for low values of P_o/P vs M has been plotted and $P_o/P_{2'}$ can be obtained directly using the value of $M_{2'}$ obtained.

Thrust Augmentation

The net thrust on a thrust augmentor is due to the difference between the internal and external integrated pressures. The internal integrated pressures is equal to the change in momentum between the bell mouth and the beginning of the mixing length (assuming constant total momentum in the constant area mixing tube length). The sum of the external forces is composed of the normal pressure forces over the bounding surface.

For steady flow from (4)*

$$PdA - Fdx = d\,(PA + \rho A v^2) \qquad \ldots\ldots(30)$$

Assuming no friction

$$PdA = d\,(PA + \rho A v^2) \qquad \ldots\ldots(31)$$

Since $\rho = \dfrac{P}{gRT}$ and $M = \dfrac{v}{\sqrt{\gamma gRT}}$

(2) becomes

$$\text{Net wall reaction} = PdA = d(PA\,[1 + \gamma M^2]\,) \qquad \ldots\ldots(32)$$

Integrating

$$\text{Net wall reaction} = P_2A_2\,(1 + \gamma M_2^2) - P_{2'}A_{2'}\,(1 + \gamma M_{2'}^2) \qquad \ldots\ldots(33)$$

Since the pressures are measured above absolute zero the above equation must be corrected for external force $\int_{2'}^{2} PdA = P_3(A-A')$ The net thrust on the augmentor is the equal to

$$F = P_2\,(A-a)(1 + \gamma M_2^2) - P_{2'}(A'-a)(1 + \gamma M_{2'}^2) + P_3(A'-A) \qquad \ldots\ldots(34)$$

A further refinement can be made by computing the net thrust on the primary jet and determining the ratio of

the two. However, in this thesis only a quantitative measurement of the effect of the various design parameters was desired so such a comparison was deemed not necessary.

TABLE I

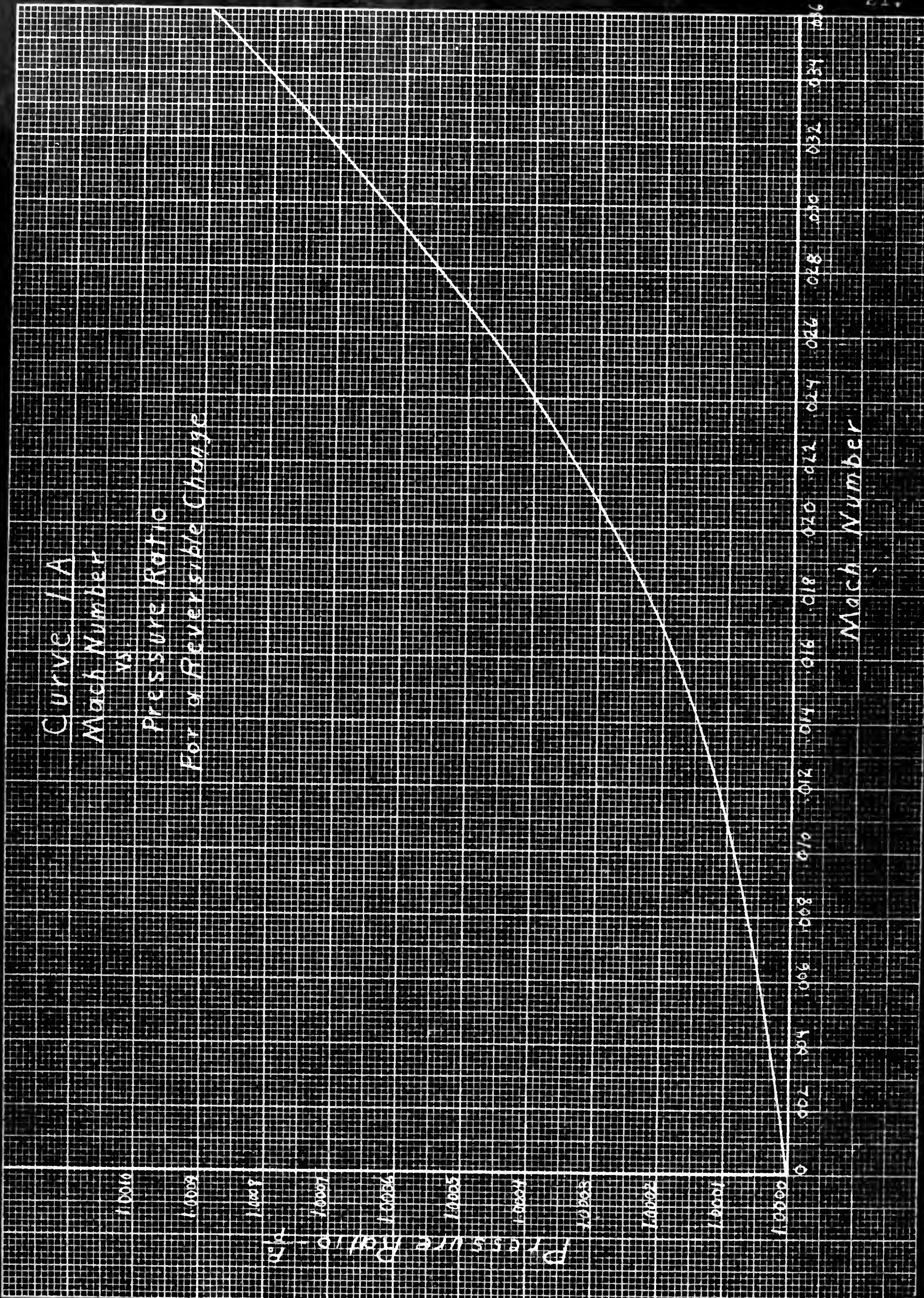
Values used in computing curves.

$\frac{P_o}{P}$	M	M	$\frac{P_o}{P}$	$\frac{W\sqrt{T_o}}{AP}$	$\frac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)}}$	$\frac{A}{A_o}$
1.0001	.01197	.17	1.0203	.1565	6.6474	
1.0002	.01699	.18	1.0228	.1658	6.3040	3.2797
1.0003	.02075	.19	1.0253	.1750	6.0021	
1.0004	.02392	.20	1.0282	.1843	5.7290	2.9650
1.0005	.02681	.21	1.0310	.1936	5.4834	
1.0006	.02934	.22	1.0342	.2029	5.2613	2.7090
1.0007	.03166	.23	1.0374	.2122	5.0603	
1.0008	.03383	.24	1.0407	.2216	4.8752	2.4968
1.0009	.03593	.25	1.0443	.2309	4.7085	
1.001	.03785	.26	1.0480	.2403	4.5539	2.3183
1.002	.05347	.27	1.0518	.2496	4.4138	
1.003	.06556	.28	1.0558	.2590	4.2833	2.1666
1.004	.07567	.29	1.0599	.2684	4.1629	
1.005	.08458	.30	1.0643	.2778	4.0517	2.0360
1.006	.09264	.31	1.0686	.2873	3.9473	
1.007	.10005	.32	1.0732	.2967	3.8519	1.9227
1.008	.10695	.33	1.0780	.3062	3.7620	
1.009	.11341	.34	1.0830	.3157	3.6784	1.8236
1.010	.11947	.35	1.0881	.3252	3.601	
1.011	.12533	.36	1.0933	.3347	3.528	1.7365
1.012	.13088	.37	1.0988	.3442	3.460	
1.013	.13621	.38	1.1044	.3538	3.396	1.6591
1.014	.14131	.39	1.1103	.3633	3.337	
1.015	.14626	.40	1.1161	.3730	3.279	1.5908
1.016	.15103	.41		.3826	3.227	
1.017	.15564	.42		.3922	3.177	
1.018	.16013	.43		.4019	3.130	
1.019	.16449	.44		.4116	3.086	
1.020	.16873	.45		.4213	3.044	
		.46		.4310	3.009	
		.47		.4408	2.968	
		.48		.4506	2.933	
		.49		.4604	2.899	
		.50		.4702	2.868	

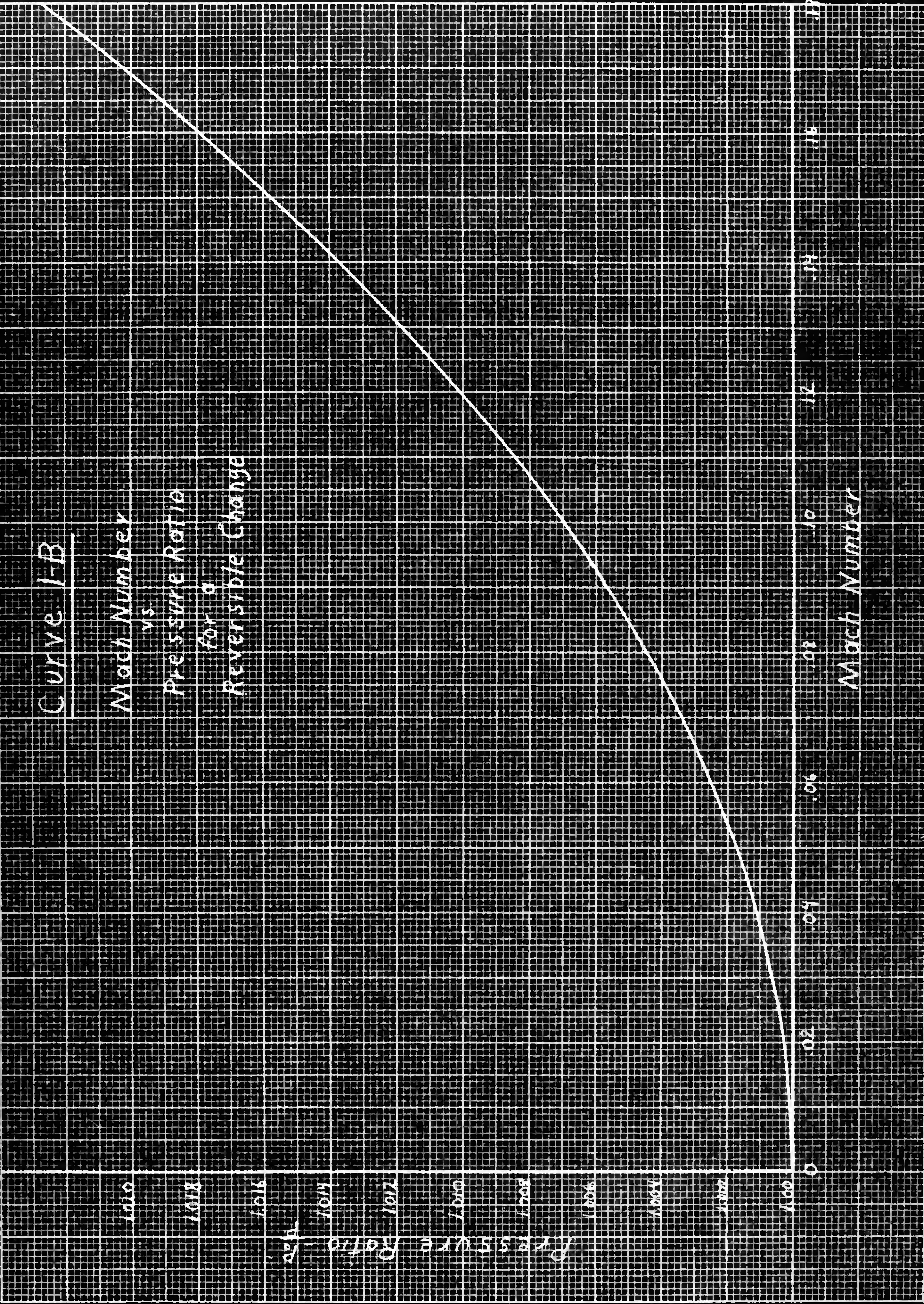
TABLE 2

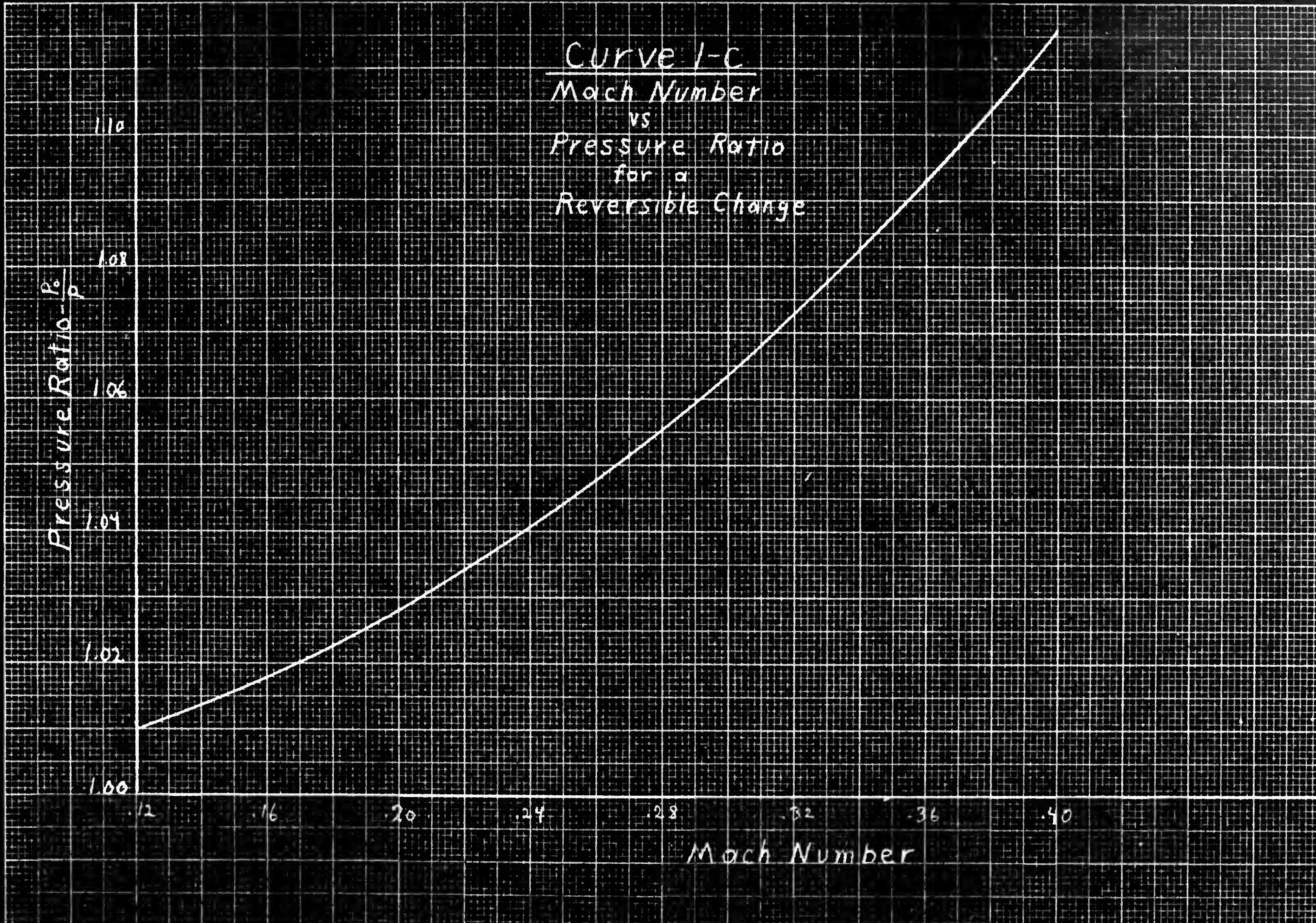
Values used in computing curves.

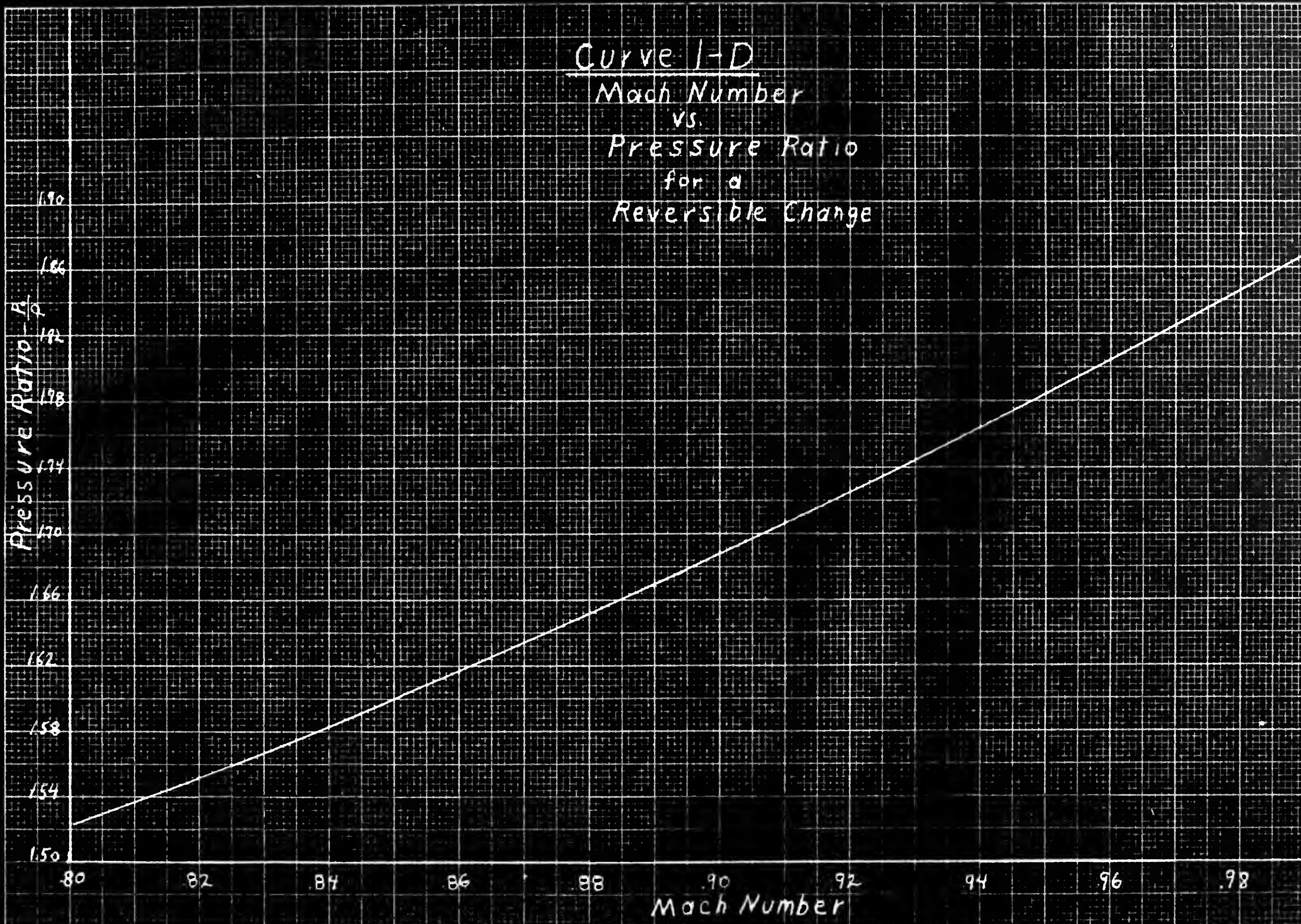
M	P_o/P	$W\sqrt{T_o}/AP$	$\dfrac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)}}$	M	A/A_o
.78	1.4931	.7578	2.4396	.392	2
.80	1.5225	.7794	2.4285	.1985	3
.82	1.5532	.8012	2.4189	.1475	4
.84	1.5853	.8231	2.4108	.1170	5
.86	1.6185	.8452	2.4039	.0975	6
.88	1.6531	.8674	2.3983	.083	7
.89	1.6708	.8786	2.3959	.073	8
.90	1.6890	.8898	2.3938	.065	9
.91	1.7073	.9011	2.3917	.0585	10
.92	1.7262	.9125	2.3898	.053	11
.93	1.7453	.9238	2.3885	.0485	12
.94	1.7649	.9352	2.3873	.0415	14
.95	1.7848	.9467	2.3862	.0363	16
.96	1.8051	.9581	2.3855	.0323	18
.97	1.8256	.9697	2.3849	.0290	20
.98	1.8468	.9813	2.3844	.0268	22
.99	1.8681	.9929	2.3839	.0242	24
1.00	1.8899	1.0046	2.3840	.0223	26
1.02	1.9349	1.0281	2.3844	.0207	28
1.04	1.9813	1.0518	2.3852	.0195	30
1.06	2.0295	1.0757	2.3867	.0145	40
1.08	2.0799	1.0998	2.3887	.0117	50
1.10	2.1315	1.1241	2.3912	.0097	60
1.12	2.1849	1.1484	2.3945		
1.14	2.2412	1.1732	2.3976		
1.16	2.2988	1.1982	2.4012		
1.18	2.3584	1.2232	2.4055		



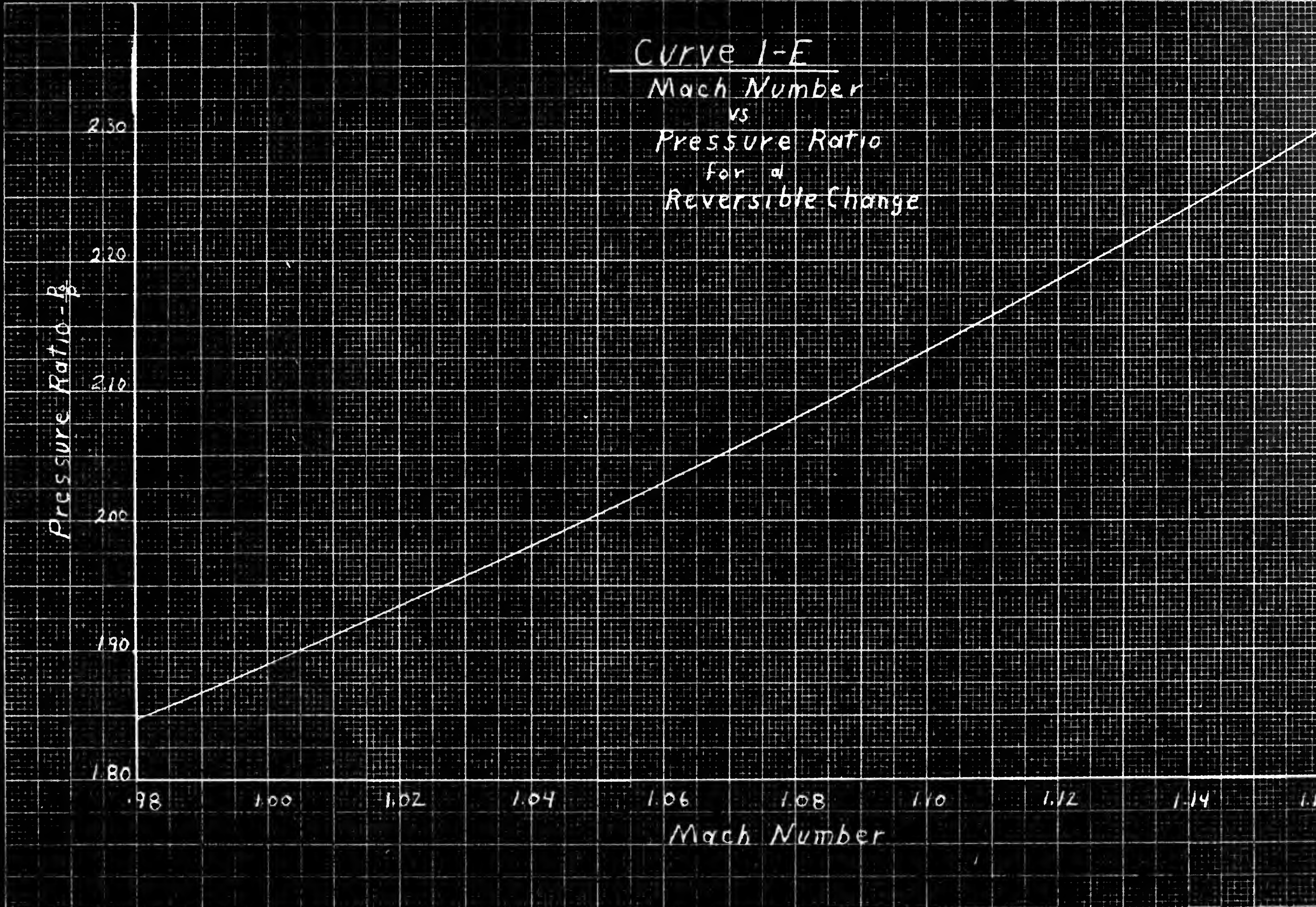
Curve IA
Mach Number
vs
Pressure Ratio
For a Reversible Change
Mach Number
0
.002
.004
.006
.008
.010
.012
.014
.016
.018
.020
.022
.024
.026
.028
.030
.032
.034
.036
Pressure Ratio
1.0000
1.0001
1.0002
1.0003
1.0004
1.0005
1.0006
1.0007
1.0008
1.0009
1.0010

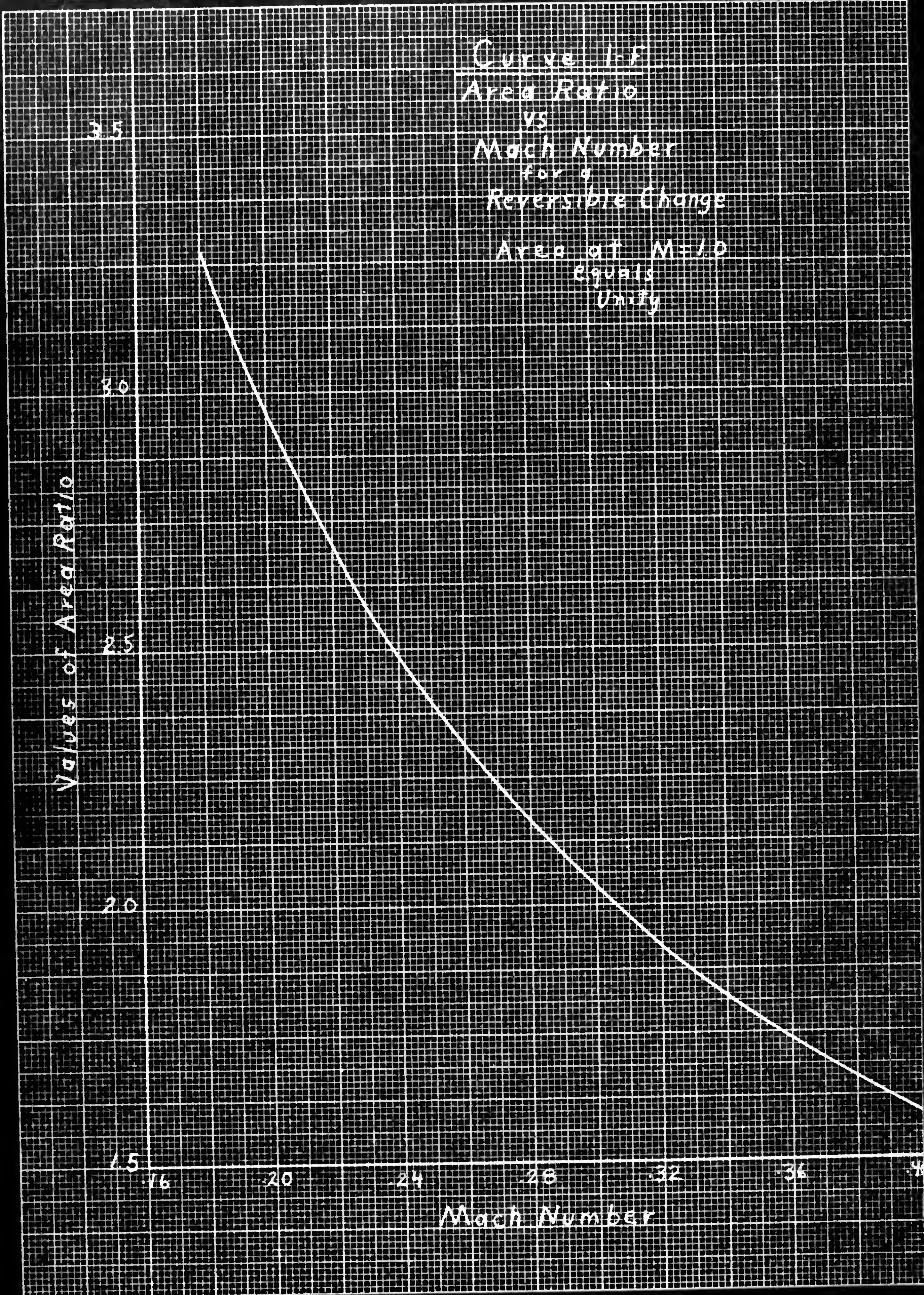

Curve I-B
Mach Number
vs.
Pressure Ratio
for a
Reversible Change
Mach Number
Pressure Ratio
1.020
1.018
1.016
1.014
1.012
1.010
1.008
1.006
1.004
1.002
1.00
0
.02
.04
.06
.08
.10
.12
.14
.16
.18

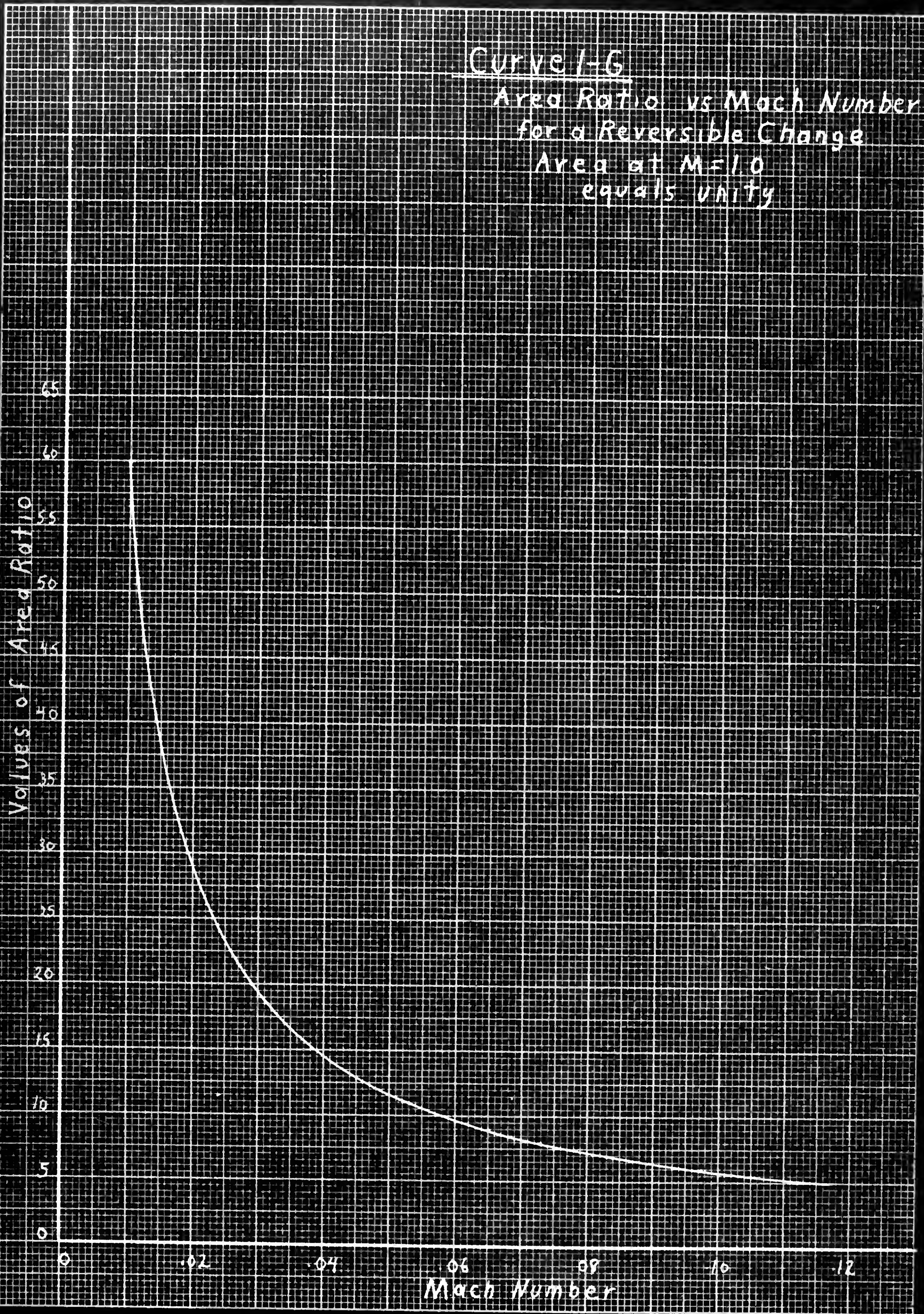

Curve I-C
Mach Number
vs
Pressure Ratio
for a
Reversible Change
1.10
1.08
1.06
1.04
1.02
1.00
Pressure Ratio - $\frac{P_0}{P}$
.12
.16
.20
.24
.28
.32
.36
.40
Mach Number

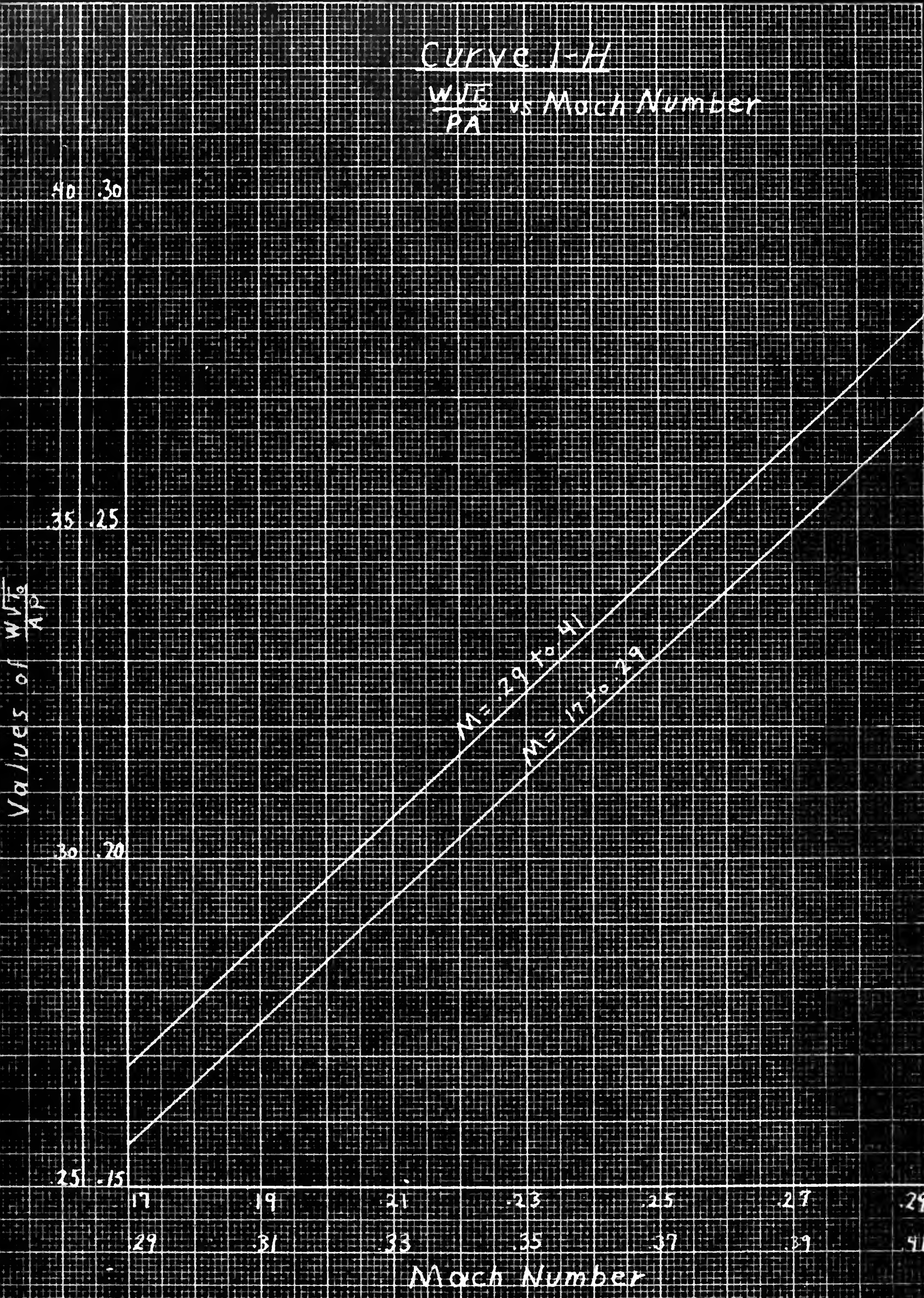


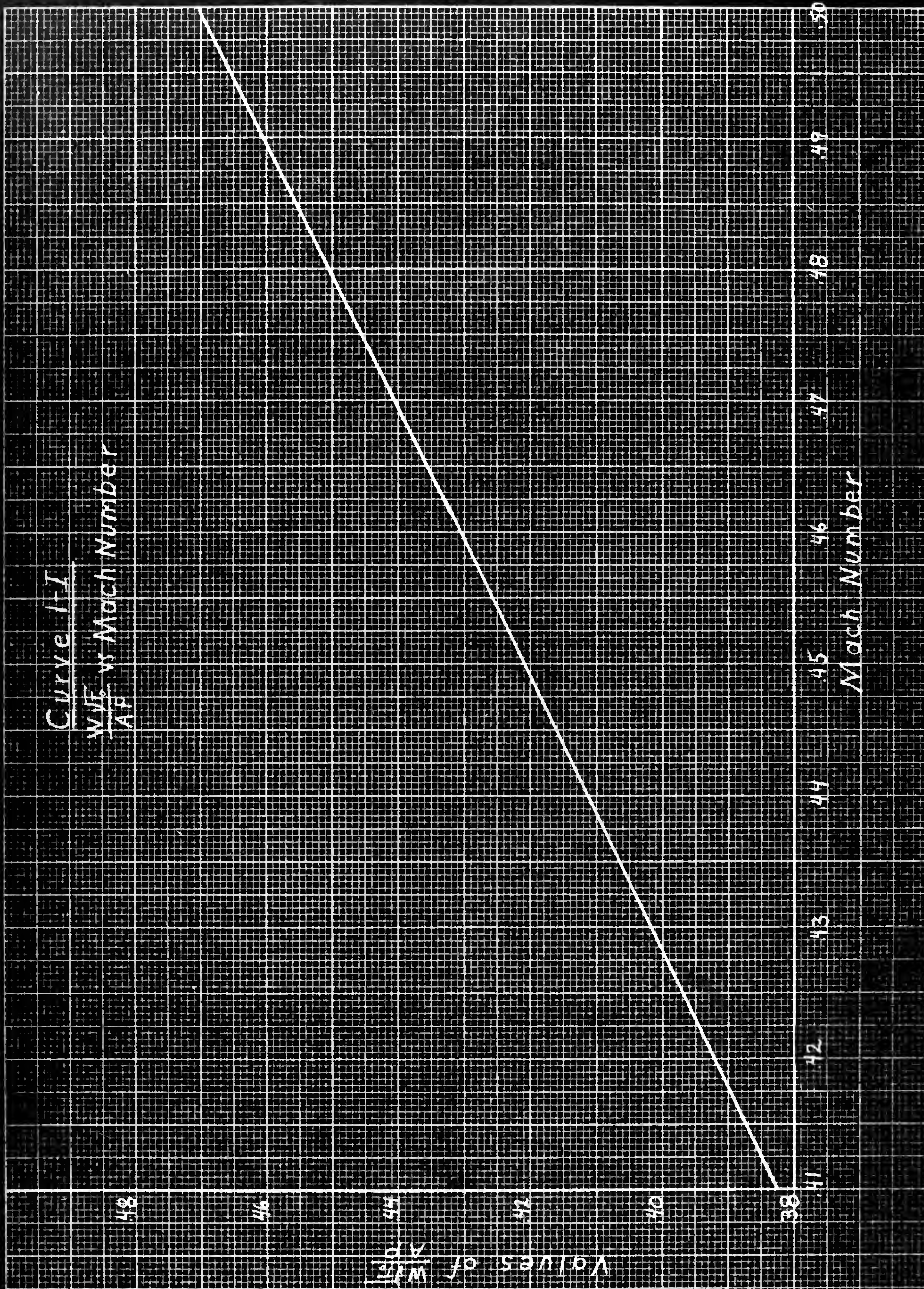
Curve I-D
Mach Number
vs.
Pressure Ratio
for a
Reversible Change
Pressure Ratio - $\frac{P}{p}$
1.50
1.54
1.58
1.62
1.66
1.70
1.74
1.78
1.82
1.86
1.90
.80
.82
.84
.86
.88
.90
.92
.94
.96
.98
Mach Number

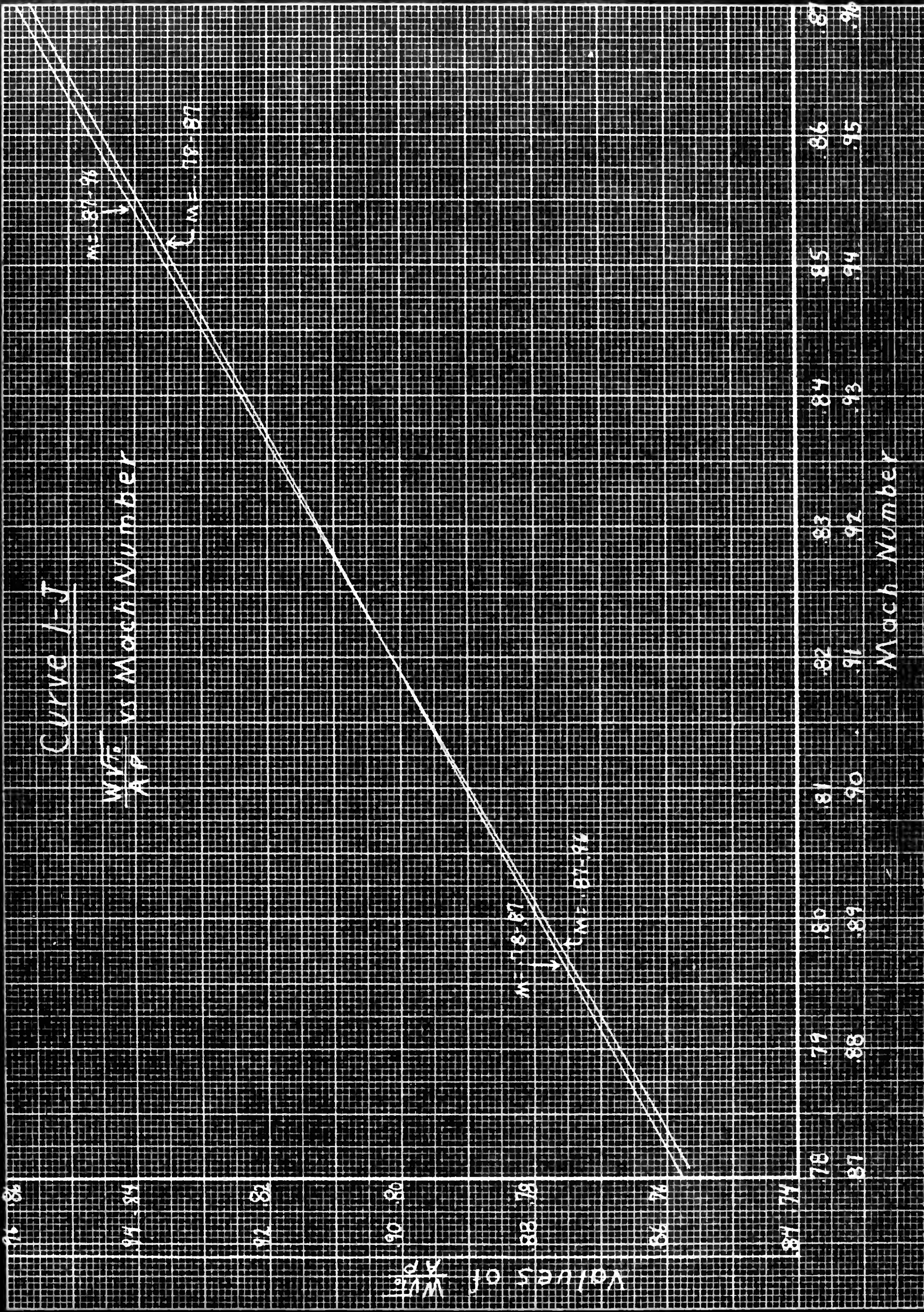

Curve 1-E
Mach Number
vs
Pressure Ratio
for a
Reversible Change
Pressure Ratio - P/p
2.30
2.20
2.10
2.00
1.90
1.80
.98
1.00
1.02
1.04
1.06
1.08
1.10
1.12
1.14
1.16
Mach Number

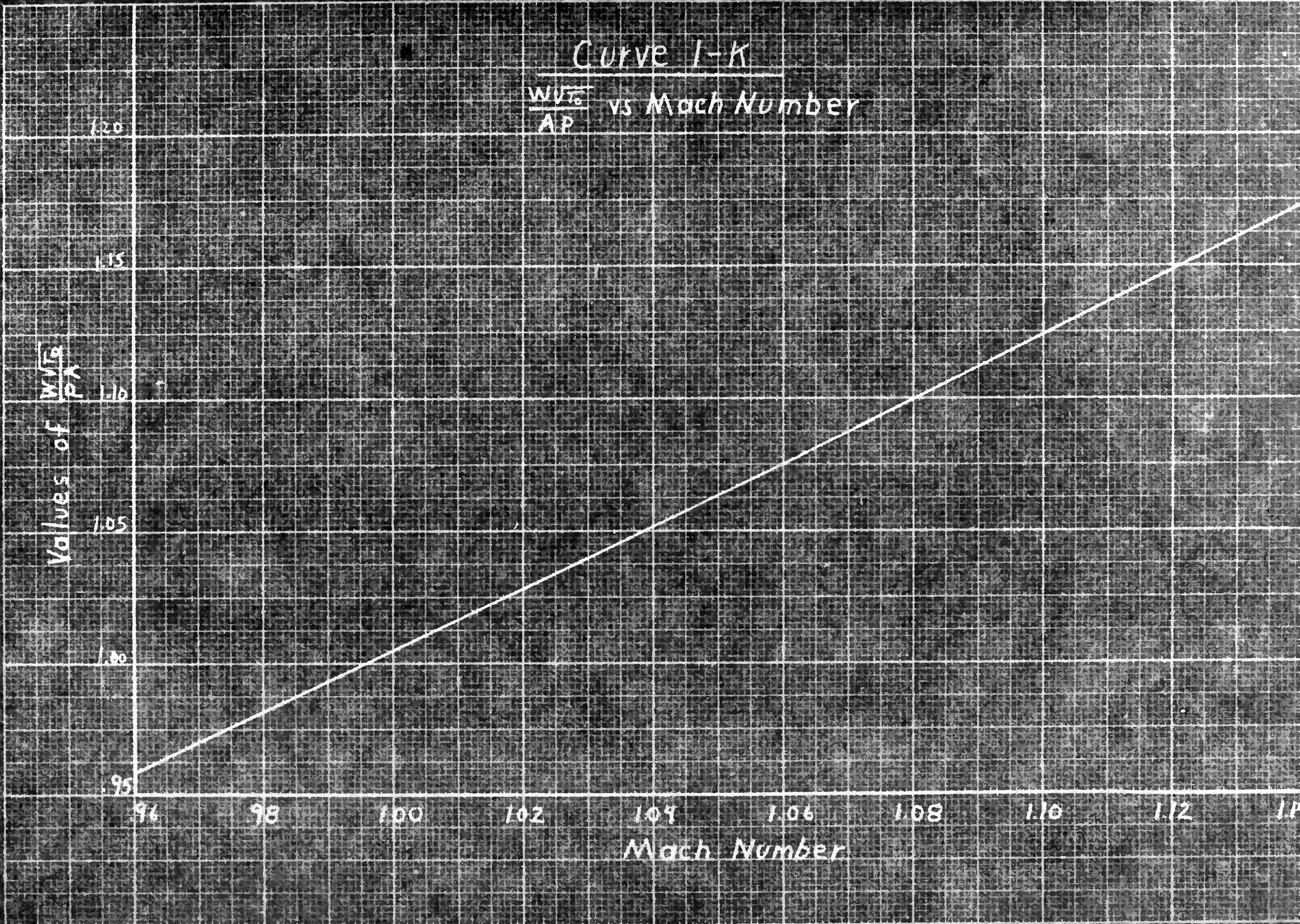

Curve I-F
Area Ratio
vs
Mach Number
for a
Reversible Change
Area at M=1.0
Equals
Unity
3.5
3.0
2.5
2.0
1.5
Values of Area Ratio
.16
.20
.24
.28
.32
.36
.40
Mach Number

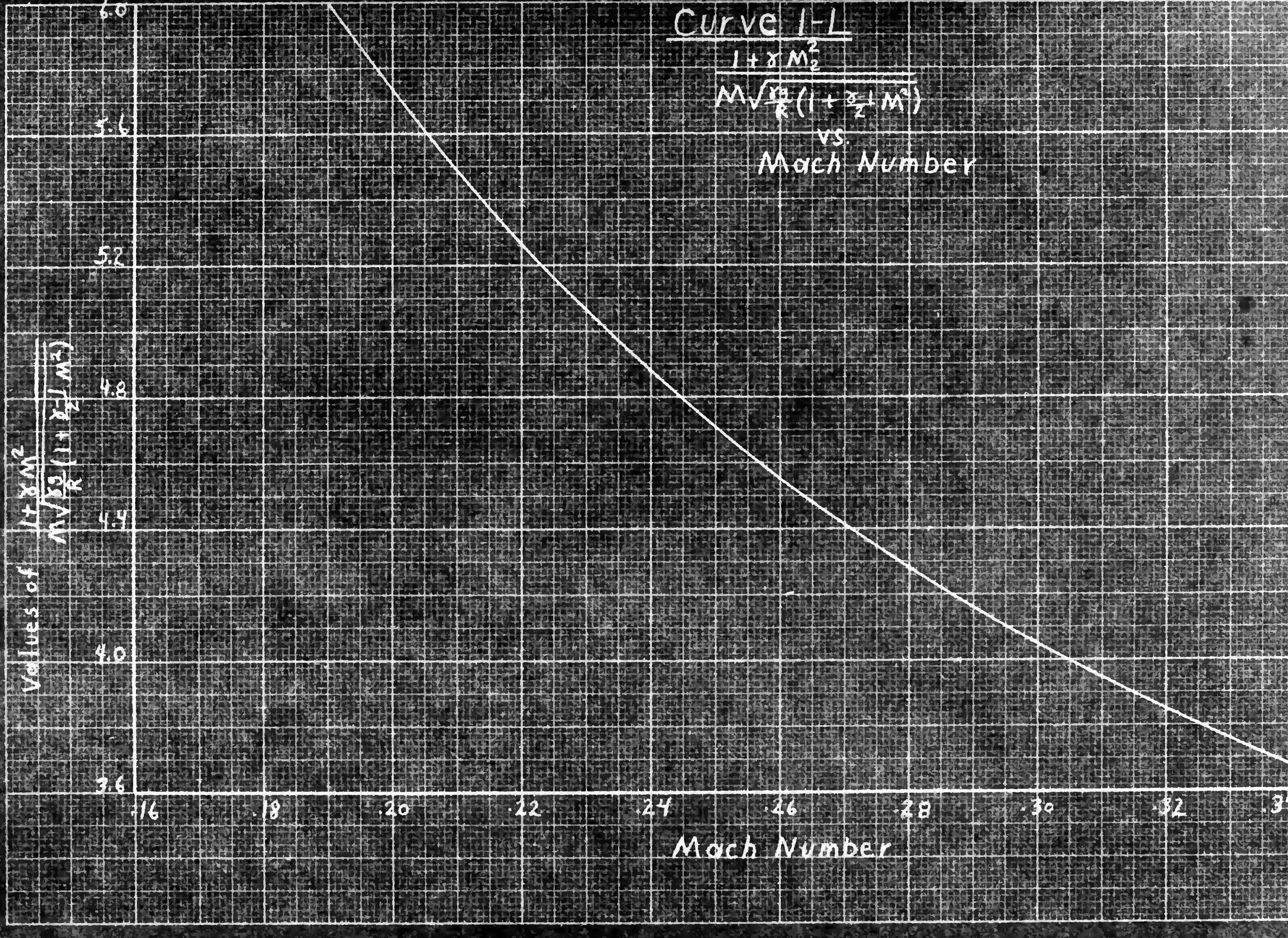

Curve I-G
Area Ratio vs Mach Number
for a Reversible Change
Area at M=1.0
equals unity
Values of Area Ratio
65
60
55
50
45
40
35
30
25
20
15
10
5
0
0
.02
.04
.06
.08
.10
.12
Mach Number

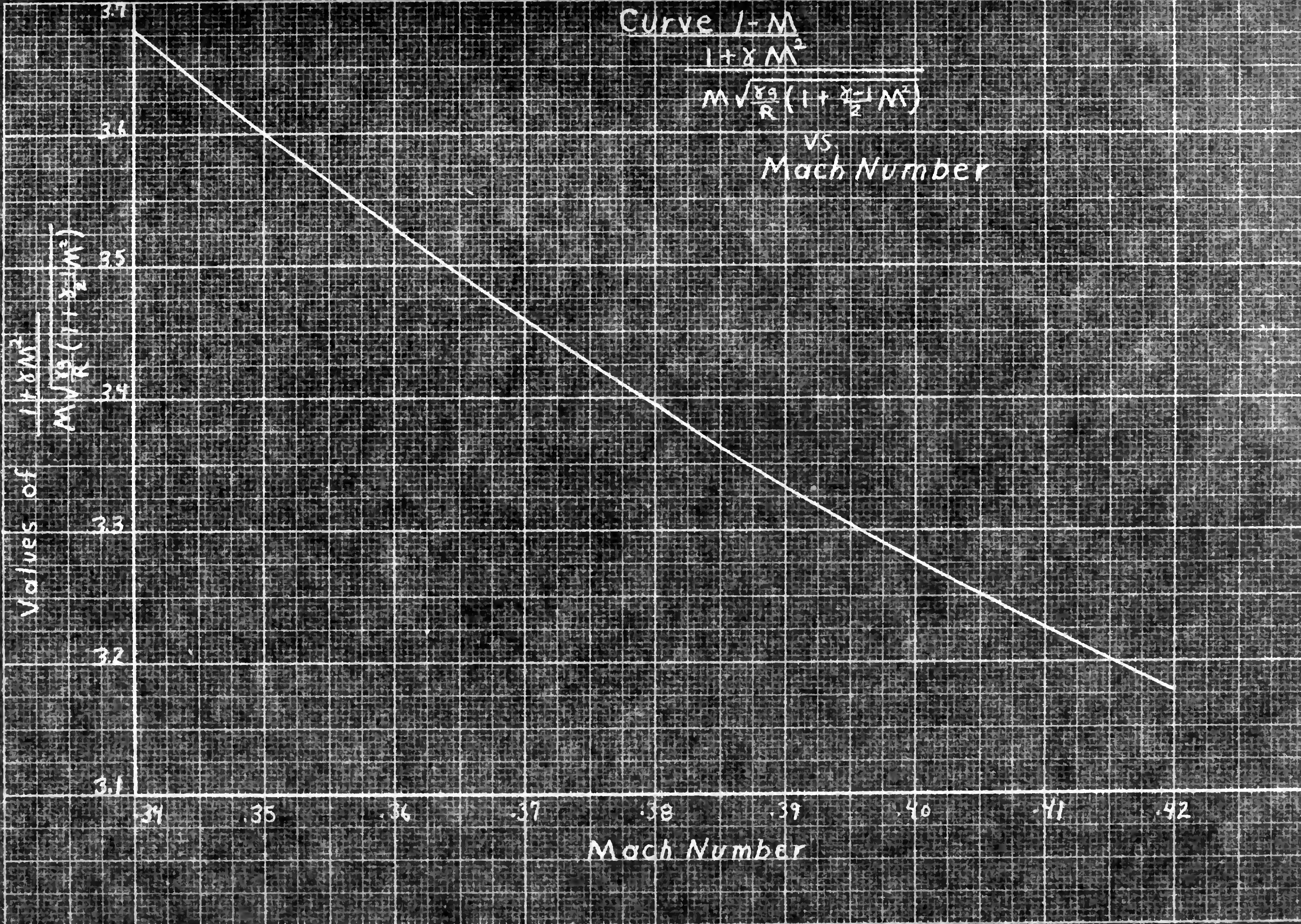

Curve I-H
$\frac{W\sqrt{T_0}}{PA}$ vs Mach Number
Values of $\frac{W\sqrt{T_0}}{AP}$
.40 .30
.35 .25
.30 .20
.25 .15
M= .29 to .41
M= .17 to .29
.17 .19 .21 .23 .25 .27 .29
.29 .31 .33 .35 .37 .39 .41
Mach Number


Curve I-I
W√T/AP vs Mach Number
Mach Number
Values of W√T/AP
.41
.42
.43
.44
.45
.46
.47
.48
.49
.50
38
40
42
44
46
48

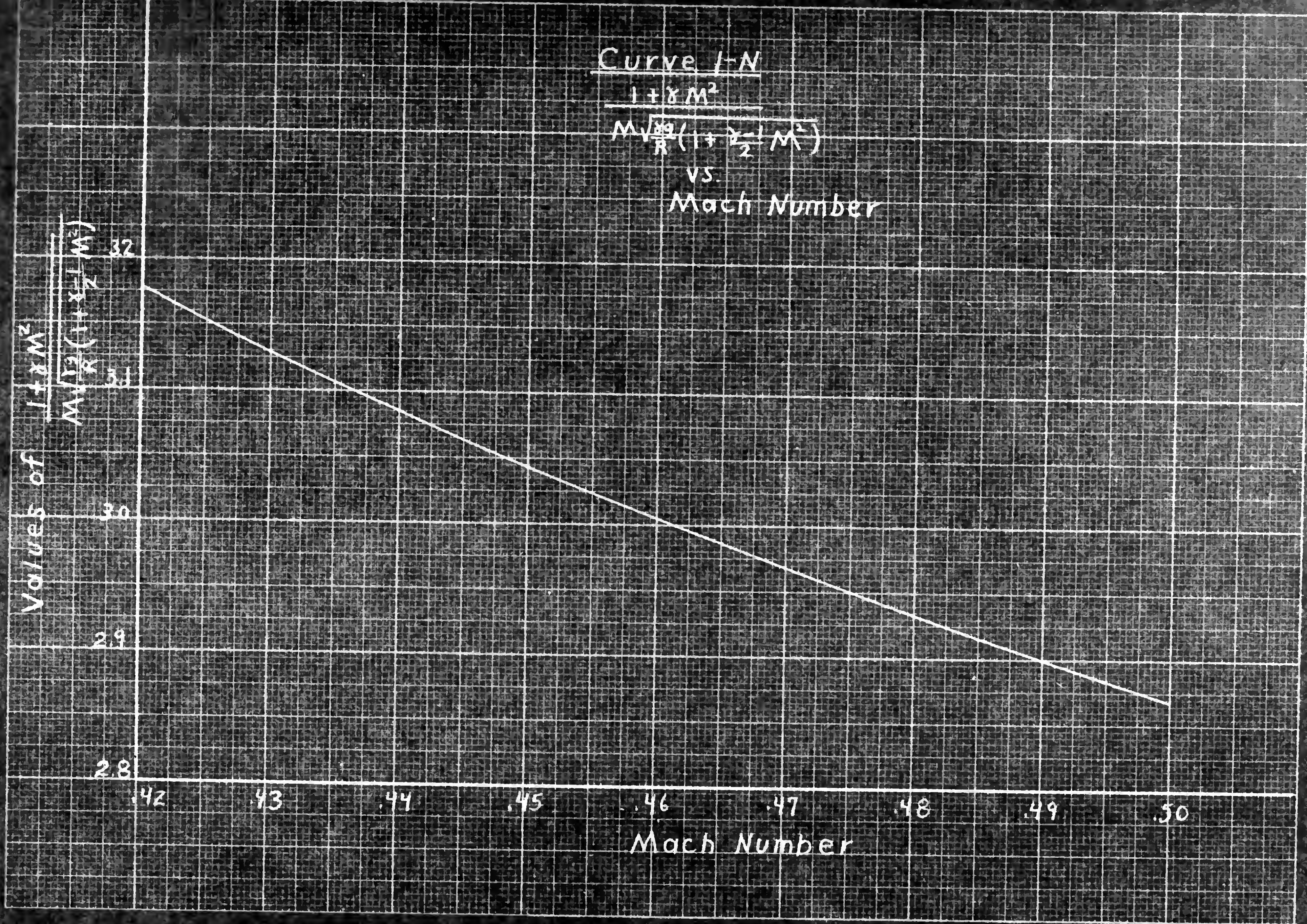

Curve I-J
$\frac{W\sqrt{T}}{AP}$ vs Mach Number
Values of $\frac{W\sqrt{T}}{AP}$
Mach Number
m = .78-.87
m = .87-.96

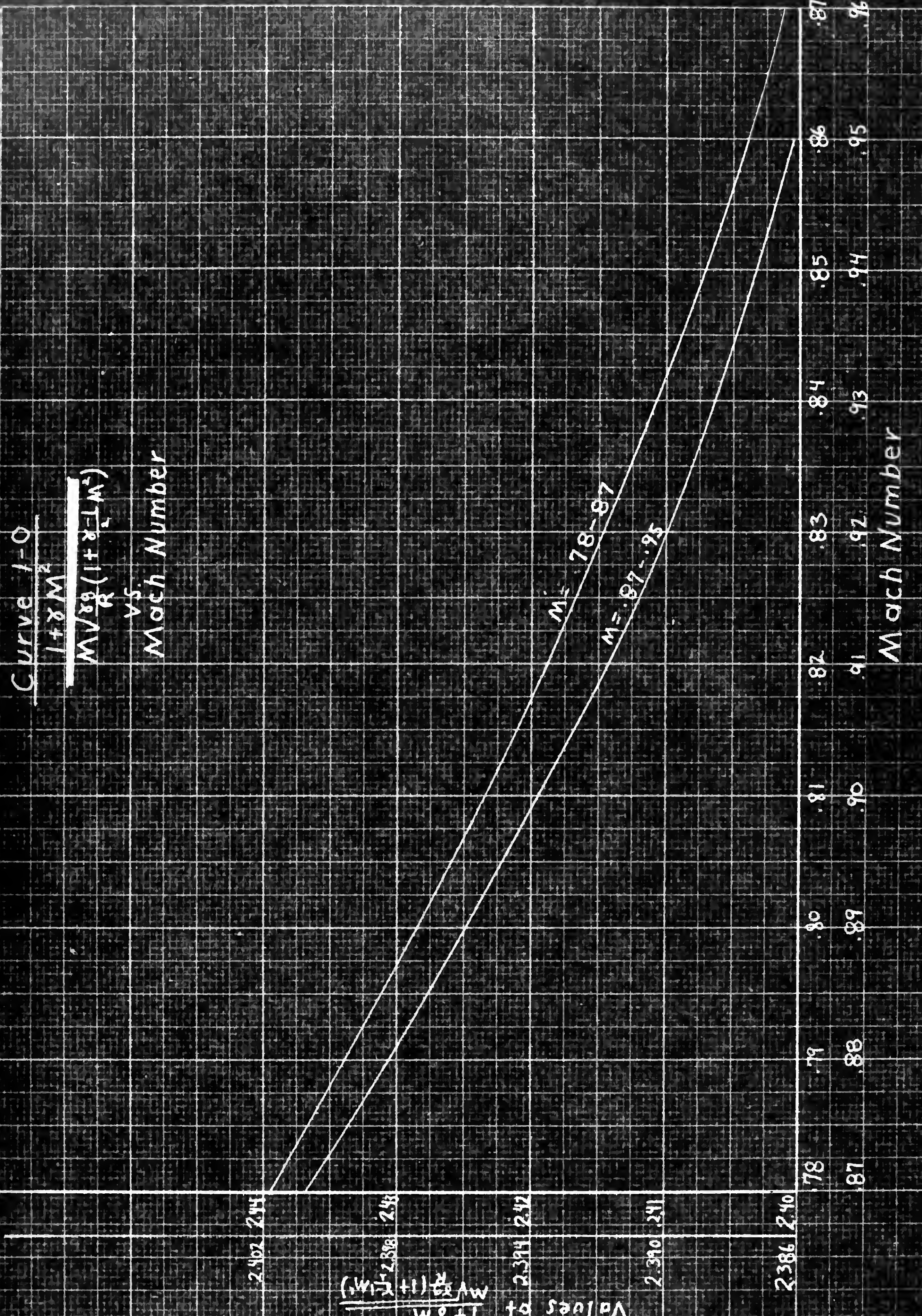

Curve I-K
$\frac{W\sqrt{T_0}}{AP}$ vs Mach Number
Values of $\frac{W\sqrt{T_0}}{PA}$
1.20
1.15
1.10
1.05
1.00
.95
.96
.98
1.00
1.02
1.04
1.06
1.08
1.10
1.12
1.14
Mach Number


Curve I-L
$\dfrac{1+\gamma M_2^2}{M\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)}}$
vs.
Mach Number
Values of $\dfrac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)}}$
6.0
5.6
5.2
4.8
4.4
4.0
3.6
.16
.18
.20
.22
.24
.26
.28
.30
.32
.34
Mach Number

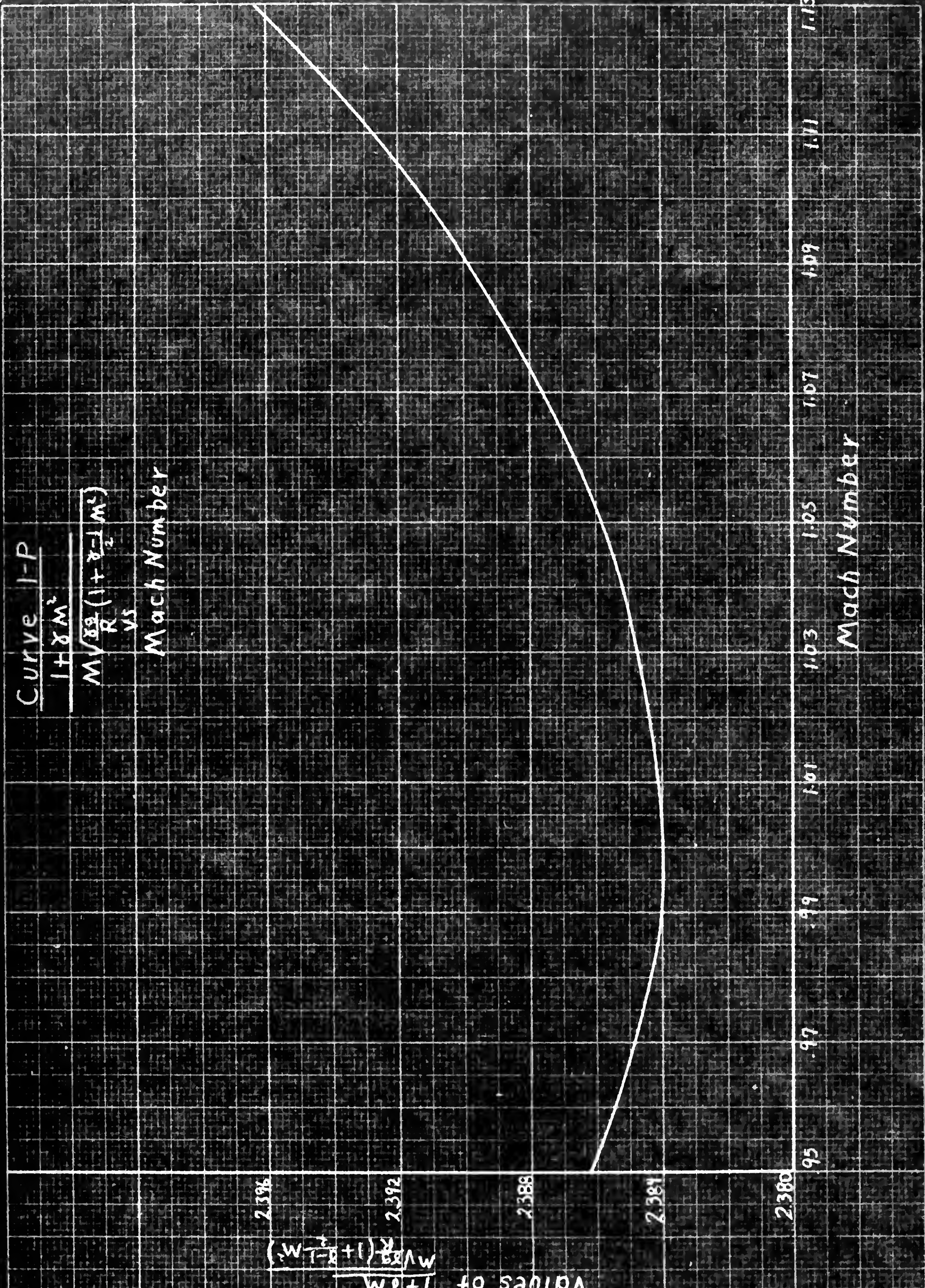


Curve 1-M
$\frac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)}$
vs.
Mach Number
Values of $\frac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)}$
3.7
3.6
3.5
3.4
3.3
3.2
3.1
.34
.35
.36
.37
.38
.39
.40
.41
.42
Mach Number


Curve I-N
$\dfrac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)}$
vs.
Mach Number
Values of $\dfrac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)}$
3.2
3.1
3.0
2.9
2.8
.42
.43
.44
.45
.46
.47
.48
.49
.50
Mach Number



Curve 1-0
$\frac{1+\gamma M^2}{M\sqrt{\gamma g}\left(1+\frac{\gamma-1}{2}M^2\right)}$
vs.
Mach Number
M= .78-.87
M= .87-.95
.78 .79 .80 .81 .82 .83 .84 .85 .86 .87
.87 .88 .89 .90 .91 .92 .93 .94 .95 .96
Mach Number
2.44 2.43 2.42 2.41 2.40
2.402 2.398 2.394 2.390 2.386
Values of $\frac{1+\gamma M^2}{M\sqrt{\gamma g}\left(1+\frac{\gamma-1}{2}M^2\right)}$


Curve 1-P
$\frac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)}$
vs
Mach Number
2.396
2.392
2.388
2.384
2.380
.95
.97
.99
1.01
1.03
1.05
1.07
1.09
1.11
1.13
Mach Number
Values of $\frac{1+\gamma M^2}{M\sqrt{\frac{\gamma g}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)}$

PART II

Calculation Procedure

In a thrust augmentor if the mixed streams do not exhaust at atmospheric pressure, there will be energy loss due to "under-expansion" or "over-expansion". Consequently, if it assumed that the exhaust is at atmospheric, then under static conditions $P_{o_2}/P_1 = P_3/P_1$, where $P_2 = P_1$.

With the above relation, it is possible to solve the thrust augmentor problem. It is not practicable to solve directly. For any given set of physical dimensions, total pressure ratio, and temperature, there is only one M_2 which will be a solution. By assuming an M_2 and using equations (13) and (16), M_3 can be computed. With M_1, M_2, M_3, and W_2/W_1, equation (26) can be used and P_3/P_1 calculated. By using various values of M_2 and solving for P_3/P_1, a plot of P_3/P_1 and P_{o_2}/P_1 vs M can be made. The point of intersection of these two curves is the solution.

After a solution has been obtained the net thrust can be calculated from equation (34):

$$F = P_2\ (A-a)(1 + \gamma M_2^2) - P_{2'}\ (A'-a)(1 + \gamma M_{2'}^2) + P_3\ (A'-A)$$

There are four fundamental variables in the thrust augmentation problem. These are T_{o_1}/T_{o_2}, P_{o_2}/P_{o_1}, A/a and A'/A, (see figure 1 for nomenclature). Since the permutations and combinations of these four variables would be practically endless, it was decided to use three values of temperature ratio, 2, 3, and 4, and three values of pressure ratio 1.5 , 1.7

and 1.9. Since the value of M_2 depends on temperature ratio, pressure ratio and A/a each of the preceding nine possible combinations of pressure and temperature ratios was computed using six values of area ratio, 5, 10, 12.5, 15, 17.5 and 20. "a" was assumed unity. Values above 20 were not used because of the physical difficulties of such a design. Using the above combinations 54 values of M_2 were calculated. These are plotted on curves (2A) (2B) (2C). With these curves, intermediate solutions can be obtained. Since the curves are very similar, interpolation and double interpolation can be used to obtain any solution in the range of values used.

It is interesting to note that while these calculations were primarily for thrust augmentation, the curves obtained are very useful in theoretical design of a constant area air ejector. Values of W_2/W_1, M_1, and M_3 are not plotted but are included in tables (3) and (4) for reference in case the preceding calculations were to be used for air ejector problems.

After values of M_2 had been calculated, the only remaining variable was A'/A. A'/A was then varied from 2 to 20 in six steps 2, 4, 8, 12, 16 and 20. In calculation of net thrust $P_{o2} = P_3$ was assumed to be 14.7. Net thrust was then calculated from equation (34)

$$F = P_2\,(A-a)(1+\gamma M_2^2) - P_{2'}\,(A'-a)(1+\gamma M_{2'}^2) + P_3(A'-A)$$

Results are tabulated in tables 5, 6, and 7.

Sample Calculation.

Since the pages of calculation were repetitive and voluminous only sample calculations are included. The original calculations will be retained in the possession of the author if reference to them is desired.

Calculation of M_2

From equations (13) and (26) a table was set up.

The following calculations were for $P_{o_1}/P_{o_2} = 1.9$, $T_{o_2}/T_{o_1} = 3.0$, $A/a = 17.5$

Detailed steps:

(1) M_2 was assumed

(2) P_{o_2}/P_1 was obtained from curves 1-B and 1-C

(3) P_{o_1}/P_1 was obtained from relation $P_{o_1}/P_1 = (P_{o_2}/P_1)(P_{o_1}/P_{o_2})$

(4) M_1 was obtained from curve 1-D and 1-E

(5) $W_1 \sqrt{T_{o_1}}/aP_1$ was obtained from curves 1-J and 1-K

(6) $W_2 \sqrt{T_{o_2}}/(A-a)\, P_2$ was obtained from curves 1-H and 1-I

(7) W_2/W_1 was calculated as follows: $\dfrac{W_2}{W_1} = \sqrt{\dfrac{T_{o_2}}{T_{o_1}}}\ \dfrac{(A-a)}{a}\ \dfrac{f'(M_2)}{f'(M_1)}$

where $f'(M_2)$ and $f'(M_1)$ are the values obtained in (5) and (6).

(8) $f(M_1)$ was obtained from curves 1-O and 1-P

(9) $f(M_2)$ was obtained from curve 1-L, 1-M, and 1-N.

(10) $f(M_3)$ was then calculated using equation (13)

(11) P_3/P_1 was calculated using equation (26)

M_2	P_{o_2}/P_1	P_{o_1}/P_1	M_1	(x) $W_1\sqrt{T_{o_1}}/aP_1$	(y) $W_2\sqrt{T_{o_2}}/(A-a)P_2$	W_2/W_1	$f(M_1)$	$f(M_2)$
.23	1.0374	1.9711	1.0352	1.0463	.2122	5.7961	2.3849	5.0603
.24	1.0407	1.9773	1.0381	1.0497	.2216	6.0332	2.3850	4.8752
.25	1.0443	1.9842	1.0410	1.0530	.2309	6.2667	2.3852	4.7085

(1) $1+W_2/W_1$	$1+W_2/W_1$	(2) T_{o_2}/T_{o_1}	(3) $(1)\times(2)$	(4) $\sqrt{(3)}$	(5) $W_2/W_1\sqrt{T_{o_2}/T_{o_1}}\ f(M_2)$	(6) $(5)+f(M_1)$
6.7961	2.9320		19.9262	4.4634	16.9337	19.3186
7.0332	3.0111		21.1777	4.6018	16.9816	19.3666
7.2667	3.0889		22.4461	4.7377	17.0357	19.4209

(7) $f(M_3) = (6)/(4)$	(8) M_3	(9) $W_3\sqrt{T_{o_3}}/AP_3$	(10) $\sqrt{(3)}$	(11) $W_2/W_1\sqrt{T_{o_2}/T_{o_1}}$	(12) $(11)/(y)$	(13) $1/(x)$
4.328	.2763	.2554	4.4639	3.3464	15.7700	.9557
4.208	.2858	.2644	4.6018	3.4833	15.7189	.9527
4.099	.2954	.2734	4.7377	3.6181	15.6696	.9497

(14) $(12)+(13)$	(15) $(9)\times(14)$	$P_3/P_1 = (10)/(15)$
16.7257	4.2717	1.0450
16.6716	4.4080	1.0440
16.6193	4.5437	1.0427

These values of P_3/P_1 when plotted vs M_2 intersects with the plot of P_{o_2}/P_1 at $M_2 = .247$

Sample Calculation of Thrust Augmentation

Using the same conditions as in the calculation of M_2, thrust augmentation per square inch of primary jet area is calculated as follows:

A'/A	M_2	$A-a/A_o$	$A'-a/A-a$	$A'-a/A_o$	$M_{2'}$	$P_o/P_{2'}$	$\frac{14.7}{P_o/P_{2'}}$
2	.247	2.426	2.061	5.00	.1168	1.0096	14.5602
4	"	"	4.182	10.1	.0581	1.0024	14.6648
8	"	"	8.424	20.4	.0285	1.00056	14.6918
12	"	"	12.667	30.7	.0190	1.00025	14.6963
16	"	"	16.909	41.0	.0140	1.00014	14.6979
20	"	"	21.152	51.3	.0114	1.00009	14.6987

$1+\gamma M_{2'}^2$	(1) $(A'-a)(1+\gamma M_{2'}^2)(P_{2'})$	(2) $P_3(A'-A)$	$\frac{14.7}{P_o/P_2}$	$1+\gamma M_2^2$
1.01903	504.468	257.25	14.0913	1.08511
1.00471	1016.637	771.75	"	"
1.00113	2044.468	1800.75	"	"
1.00050	3073.062	2829.75	"	"
1.00027	4101.821	3858.75	"	"
1.00018	5130.769	4887.75	"	"

(3) $(A-a)(P_2)(1+\gamma M_2^2)$	Augmentation = (2) + (3) - (1)
252.295	5.077
"	7.408
"	8.577
"	8.983
"	9.224
"	9.276

% Augmentaion of A'/A = 20
54.7
79.9
92.5
96.8
99.4

TABLE 3

Results

A/a	P_{o2}/P_{o1}	T_{o1}/T_{o2}	W_2/W_1	M_3	M_2	P_o/P_2
5.0	1.5	2.0	1.9108	.3996	.3035	1.066
10.0	"	"	3.5520	.2980	.2443	1.042
12.5	"	"	4.2498	.2715	.2273	1.0366
15.0	"	"	4.8106	.2483	.2100	1.0310
17.5	"	"	5.3388	.2308	.1970	1.0273
20.0	"	"	5.9433	.2100	.1900	1.0254
5.0	"	3.0	2.2391	.3986	.2883	1.0593
10.0	"	"	4.1661	.2941	.2330	1.0383
12.5	"	"	4.9822	.2670	.2167	1.0331
15.0	"	"	5.6811	.2454	.202	1.0285
17.5	"	"	6.6008	.2272	.189	1.0250
20.0	"	"	6.9225	.2140	.180	1.0230
5.0	"	4.0	2.4093	.3629	.266	1.0502
10.0	"	"	4.6403	.2921	.224	1.0354
12.5	"	"	5.5884	.2659	.210	1.0310
15.0	"	"	6.2899	.2414	.193	1.0262
17.5	"	"	7.1560	.2183	.186	1.0243
20.0	"	"	7.9539	.2163	.179	1.0225
5.0	1.7	2.0	1.8940	.4582	.355	1.0906
10.0	"	"	3.4832	.3409	.282	1.0564
12.5	"	"	4.1369	.3095	.260	1.0479
15.0	"	"	4.7526	.2858	.244	1.0420
17.5	"	"	5.3640	.2697	.233	1.0384
20.0	"	"	5.7665	.2591	.2165	1.0330

TABLE 4

Results

A/a	P_{o_2}/P_{o_1}	T_{o_1}/T_{o_2}	W_2/W_1	M_3	M_2	P_o/P_2
5.0	1.7	3.0	2.1841	.4527	.331	1.0786
10.0	"	"	4.0542	.3351	.2665	1.0504
12.5	"	"	4.8686	.3055	.249	1.0440
15.0	"	"	5.6355	.2820	.235	1.039
17.5	"	"	6.2528	.2652	.223	1.0352
20.0	"	"	6.8633	.2480	.210	1.031
5.0	1.7	4.0	2.3749	.4472	.309	1.0682
10.0	"	"	4.5213	.3333	.2565	1.0466
12.5	"	"	5.4147	.3026	.239	1.0404
15.0	"	"	6.3070	.2818	.228	1.0365
17.5	"	"	7.0667	.2632	.216	1.033
20.0	"	"	7.6224	.2440	.2015	1.0286
5.0	1.9	2.0	1.8819	.5027	.399	1.1156
10.0	"	"	3.4361	.3760	.314	1.0762
12.5	"	"	4.0868	.3420	.290	1.0598
15.0	"	"	4.6924	.3167	.272	1.0526
17.5	"	"	5.2314	.2954	.256	1.0466
20.0	"	"	5.7493	.2790	.2435	1.0420
5.0	1.9	3.0	2.1713	.4999	.372	1.0996
10.0	"	"	4.0385	.3727	.298	1.0635
12.5	"	"	4.8158	.3386	.278	1.0545
15.0	"	"	5.5289	.3122	.2607	1.0482
17.5	"	"	6.1967	.2925	.247	1.0432
20.0	"	"	6.8382	.2865	.236	1.0394
5.0	1.9	4.0	2.3488	.4635	.345	1.0856
10.0	"	"	4.4306	.3668	.2835	1.0572
12.5	"	"	5.3144	.3335	.2645	1.0497
15.0	"	"	6.1222	.3082	.2492	1.0446
17.5	"	"	6.9118	.2894	.238	1.040
20.0	"	"	7.6432	.2735	.228	1.0367

TABLE 5

Results

Thrust Augmentation lbs per in^2 of primary jet.

$P_{o_1}/P_{o_2} = 1.5$

$T_{o_1}/T_{o_2} = 2.0$

$\frac{A'}{A}$	$\frac{A}{a} = 5.0$	$\frac{A}{a} = 10.0$	$\frac{A}{a} = 12.5$	$\frac{A}{a} = 15.0$	$\frac{A}{a} = 17.5$	$\frac{A}{a} = 20.0$
2	1.943	2.658	3.064	3.158	3.198	3.449
4	2.730	4.006	4.417	4.612	4.938	5.060
8	3.100	4.609	5.090	5.366	5.558	5.998
12	3.215	4.805	5.341	5.636	5.825	6.271
16	3.285	4.929	5.430	5.748	5.922	6.417
20	3.300	4.954	5.534	5.775	6.020	6.435

$T_{o_1}/T_{o_2} = 3.0$

2	1.771	2.531	2.762	2.953	2.810	3.070
4	2.485	3.670	4.065	4.328	4.269	4.459
8	2.817	4.223	4.696	5.018	5.075	5.343
12	2.927	4.416	4.882	5.217	5.394	5.615
16	2.987	4.512	4.988	5.379	5.501	5.694
20	3.012	4.569	5.040	5.415	5.609	5.775

$T_{o_1}/T_{o_2} = 4.0$

2	1.407	2.333	2.610	2.675	2.897	3.141
4	2.101	3.396	3.804	3.956	4.197	4.530
8	2.407	3.921	4.434	4.549	4.994	5.345
12	2.517	4.090	4.631	4.766	5.243	5.617
16	2.574	4.184	4.717	4.845	5.312	5.696
20	2.587	4.250	4.777	4.914	5.424	5.777

TABLE 6

Results

Thrust Augmentation

$P_{o_1}/P_{o_2} = 1.7$

$T_{o_1}/T_{o_2} = 2.0$

$\frac{A'}{A}$	$\frac{A}{a} = 5.0$	$\frac{A}{a} = 10.0$	$\frac{A}{a} = 12.5$	$\frac{A}{a} = 15.0$	$\frac{A}{a} = 17.5$	$\frac{A}{a} = 20.0$
2	2.617	3.675	3.893	4.268	4.519	4.496
4	3.664	5.355	5.725	6.133	6.622	6.634
8	4.128	6.061	6.631	7.160	7.679	7.704
12	4.297	6.314	6.891	7.472	8.037	8.128
16	4.363	6.463	7.071	7.607	8.011	8.299
20	4.418	6.512	7.114	7.750	8.316	8.386

$T_{o_1}/T_{o_2} = 3.0$

$\frac{A'}{A}$	$\frac{A}{a} = 5.0$	$\frac{A}{a} = 10.0$	$\frac{A}{a} = 12.5$	$\frac{A}{a} = 15.0$	$\frac{A}{a} = 17.5$	$\frac{A}{a} = 20.0$
2	2.289	3.269	3.603	3.902	4.135	4.311
4	3.216	4.730	5.344	5.783	6.060	6.270
8	3.639	5.438	6.068	6.619	7.027	7.285
12	3.779	5.657	6.344	6.946	7.349	7.596
16	3.849	5.788	6.505	7.106	7.566	7.762
20	3.880	5.806	6.543	7.217	7.661	7.844

$T_{o_1}/T_{o_2} = 4.0$

$\frac{A'}{A}$	$\frac{A}{a} = 5.0$	$\frac{A}{a} = 10.0$	$\frac{A}{a} = 12.5$	$\frac{A}{a} = 15.0$	$\frac{A}{a} = 17.5$	$\frac{A}{a} = 20.0$
2	2.017	3.051	3.377	3.759	3.883	3.907
4	2.851	4.402	4.929	5.453	5.742	5.726
8	3.212	5.047	5.627	6.310	6.617	6.699
12	3.336	5.271	5.896	6.601	6.953	7.051
16	3.403	5.414	6.029	6.738	7.079	7.153
20	3.435	5.443	6.122	6.807	7.210	7.251

TABLE 7

Results

Thrust Augmentation

$P_{o1}/P_{o2} = 1.9$

$T_{o1}/T_{o2} = 2.0$

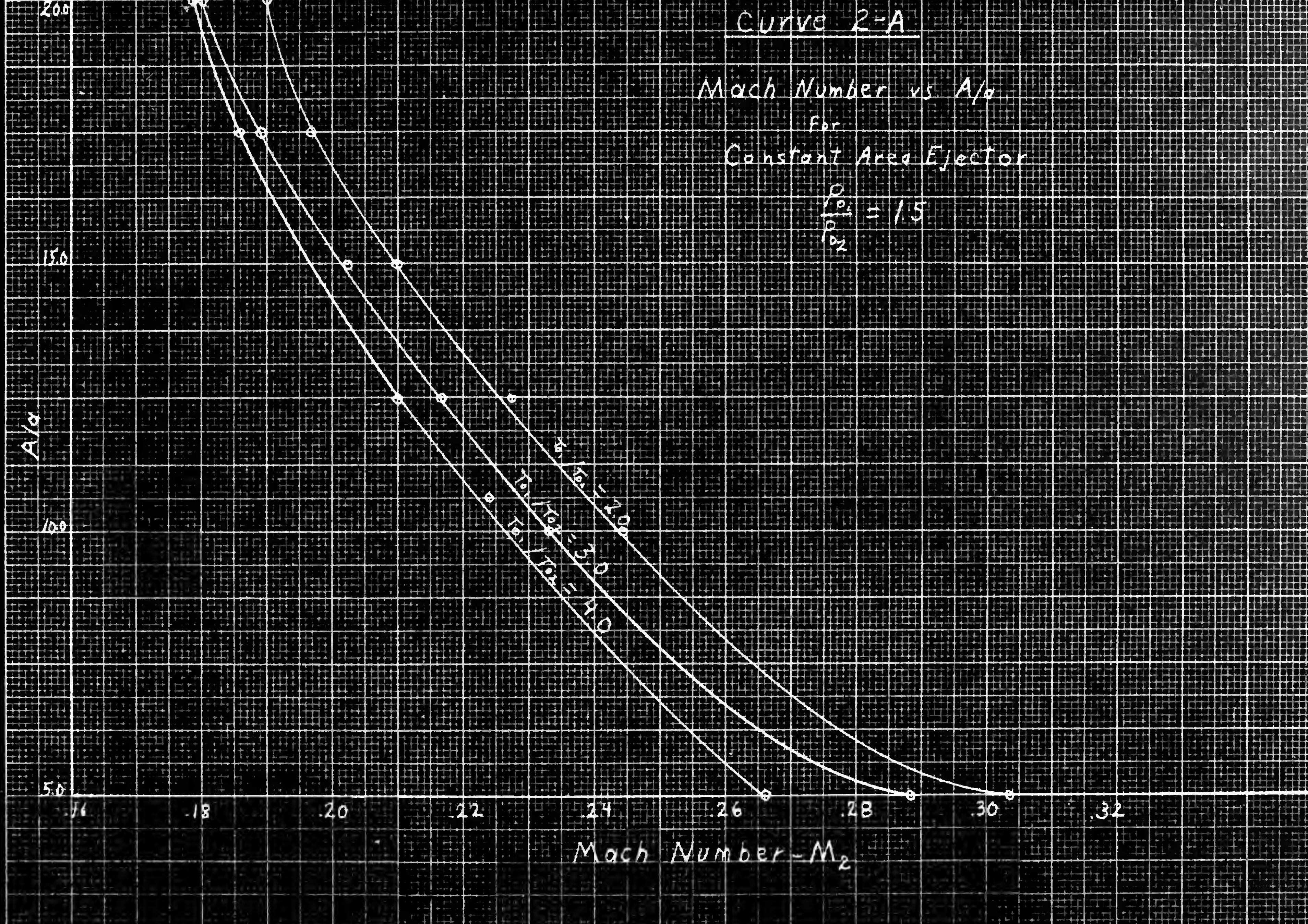
$\frac{A'}{A}$	$\frac{A}{a} = 5.0$	$\frac{A}{a} = 10.0$	$\frac{A}{a} = 12.5$	$\frac{A}{a} = 15.0$	$\frac{A}{a} = 17.5$	$\frac{A}{a} = 20.0$
2	3.772	4.522	4.826	5.187	5.409	5.681
4	5.319	6.469	7.164	7.602	7.902	8.209
8	5.990	7.401	8.130	8.734	9.139	9.576
12	6.227	7.690	8.495	9.138	9.564	10.028
16	6.342	7.875	8.648	9.343	9.812	10.316
20	6.393	7.952	8.781	9.449	9.936	10.375

$T_{o1}/T_{o2} = 3.0$

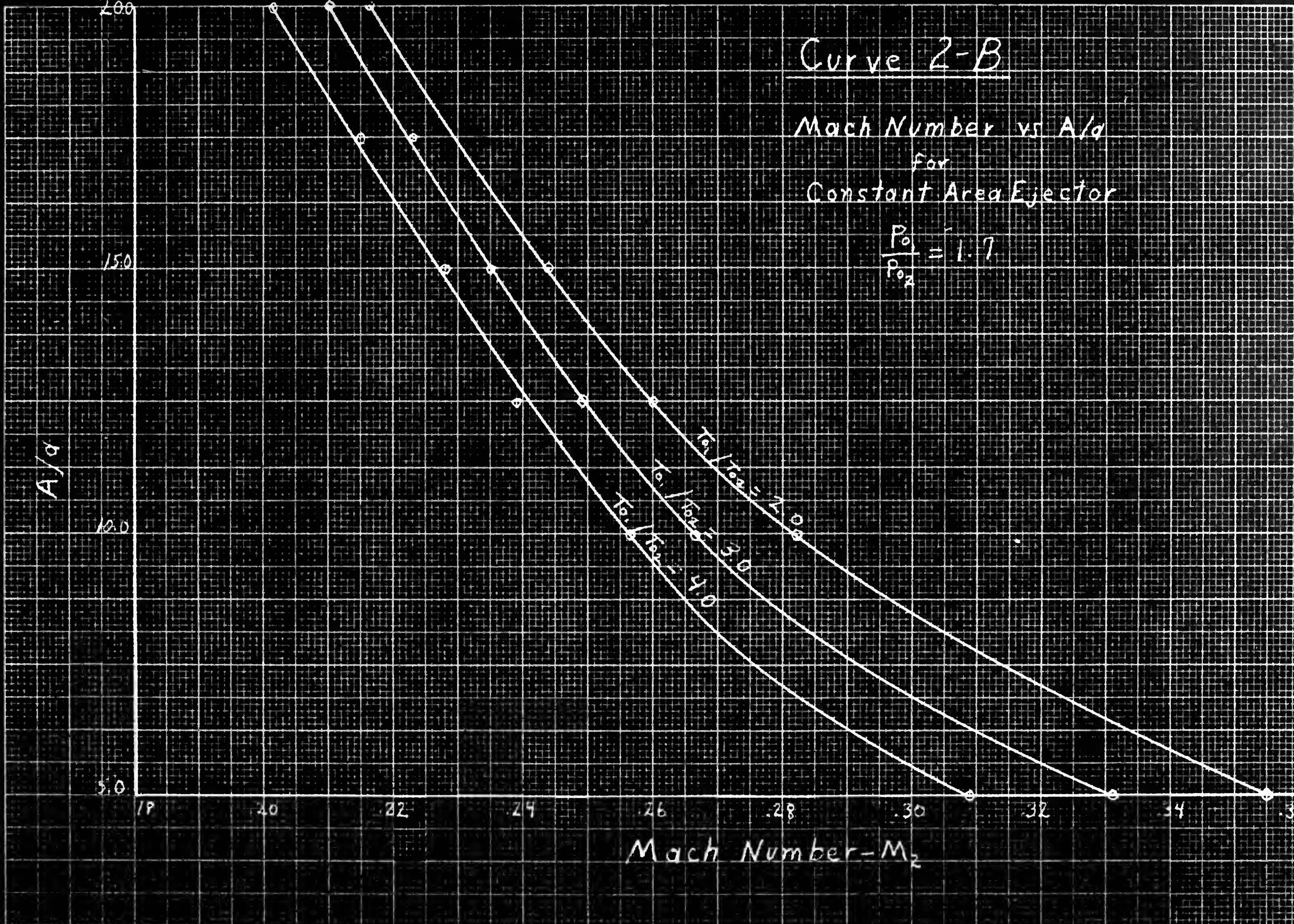
2	2.829	4.006	4.587	4.780	5.077	5.313
4	3.992	5.809	6.677	7.056	7.408	7.792
8	4.499	6.673	7.587	8.066	8.577	9.048
12	4.675	6.951	7.896	8.446	8.983	9.489
16	4.760	7.106	8.088	8.623	9.224	9.680
20	4.799	7.196	8.159	8.765	9.276	9.814

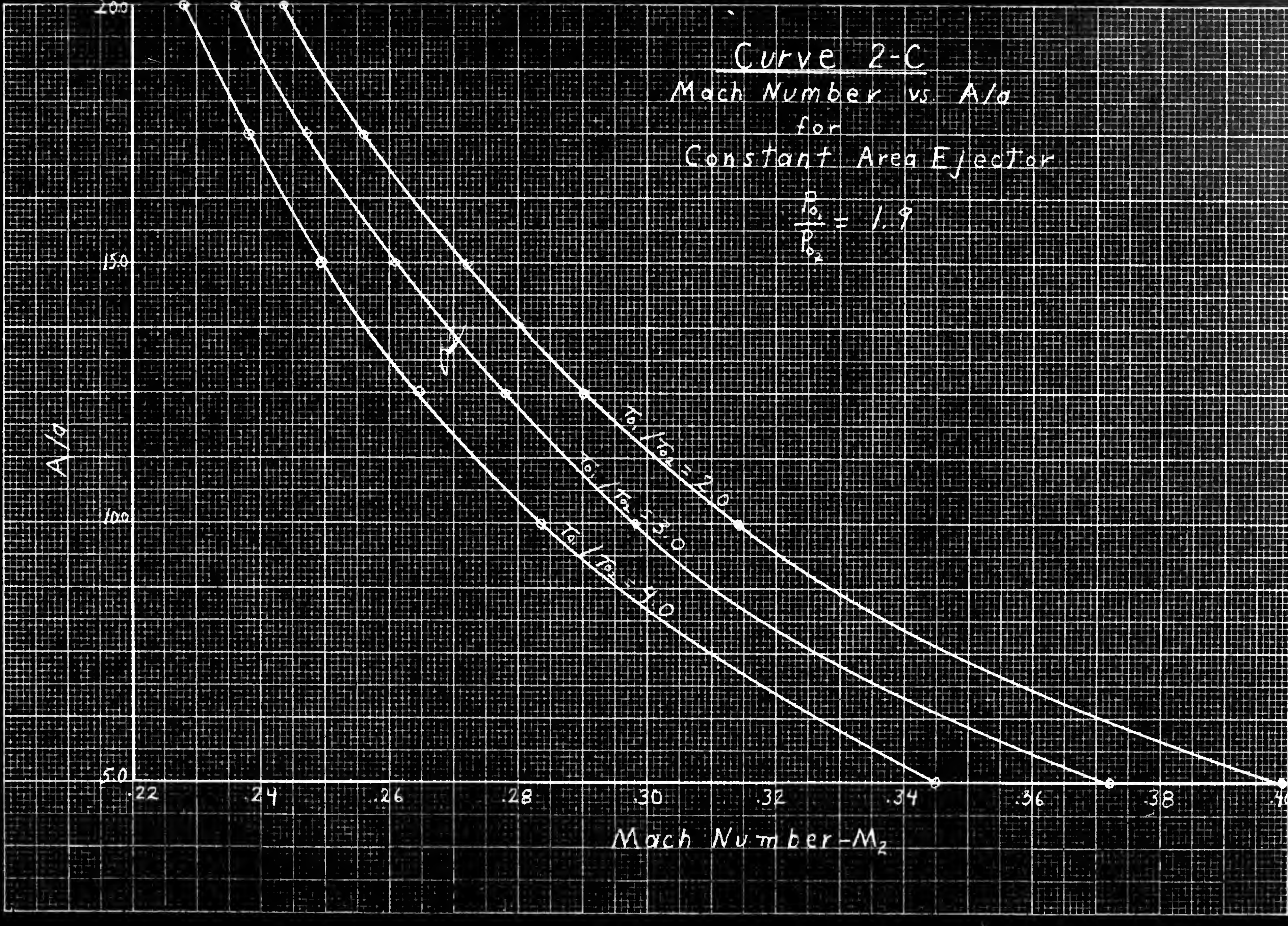
$T_{o1}/T_{o2} = 4.0$

2	2.462	3.644	4.068	4.393	4.675	4.926
4	3.457	5.234	5.837	6.363	6.936	7.386
8	3.915	6.081	6.810	7.428	8.011	8.506
12	4.069	6.354	7.104	7.742	8.398	8.880
16	4.145	6.484	7.273	7.958	8.564	9.098
20	4.185	6.578	7.391	8.002	8.682	9.191



Curve 2-A
Mach Number vs A/a
For
Constant Area Ejector
$\frac{P_{01}}{P_{02}} = 15$
$T_{01}/T_{02} = 2.0$
$T_{01}/T_{02} = 3.0$
$T_{01}/T_{02} = 4.0$
A/a
20.0
15.0
10.0
5.0
.16
.18
.20
.22
.24
.26
.28
.30
.32
Mach Number - M_2


Curve 2-B
Mach Number vs A/a
For
Constant Area Ejector
$\frac{P_{o_1}}{P_{o_2}} = 1.7$
$T_{o_1}/T_{o_2} = 2.0$
$T_{o_1}/T_{o_2} = 3.0$
$T_{o_1}/T_{o_2} = 4.0$
A/a
20.0
15.0
10.0
5.0
.18
.20
.22
.24
.26
.28
.30
.32
.34
Mach Number - M_2


Curve 2-C
Mach Number vs. A/a
for
Constant Area Ejector
$\frac{P_{o_1}}{P_{o_2}} = 1.9$
$T_{o_1}/T_{o_2} = 2.0$
$T_{o_1}/T_{o_2} = 3.0$
$T_{o_1}/T_{o_2} = 4.0$
A/a
20.0
15.0
10.0
5.0
.22
.24
.26
.28
.30
.32
.34
.36
.38
.40
Mach Number-M_2

Discussion of Results

From tables 5, 6 and 7 it is fairly obvious that in all cases the augmentation follows the same general pattern. Plots were made of the effect of the various variables, holding two variables constant and plotting a series of curves of the third variable with the fourth value as the abscissas. These are curves 2-D to 2-G inclusive.

From these curves it is seen that an increase in pressure ratio increases the augmentation. An increase in temperature ratio decreases the augmentation. An increase in mixing throat area ratio increases the augmentation, and an increase in the bell mouth area to mixing length area increases the augmentation.

The following percentage values are representative. A number of percentage calculations were made and they all were within close range of the values indicated.

Effect of Pressure Ratio

$T_{o_1}/T_{o_2} = 2.0$ $A/a = 15.0$

P_o/P_o	1.5	1.7
A'/A	% of 1.9	% of 1.9
2	60.9	82.3
4	60.7	80.7
8	61.4	82.0
12	61.7	81.8
16	61.5	81.4
20	61.1	82.0

Effect of Temperature Ratio

T_{o_1}/T_{o_2}	3	4
A'/A	% of $T_{o_1}/T_{o_2} = 2$	% of $T_{o_1}/T_{o_2} = 2$
2	91.4	88.1
4	94.3	88.9
8	92.4	88.1
12	93.0	88.3
16	93.4	88.6
20	93.1	87.8

$P_{o_1}/P_{o_2} = 1.7$

$A/a = 15.0$

Effect of A'/A on Thrust Augmentation

A'/A	% of $A'/A = 20$
2	54.1
4	80.1
8	91.7
12	96.2
16	98.5
20	

$P_{o_1}/P_{o_2} = 1.7$

$A/a = 15.0$

$T_{o_1}/T_{o_2} = 3.0$

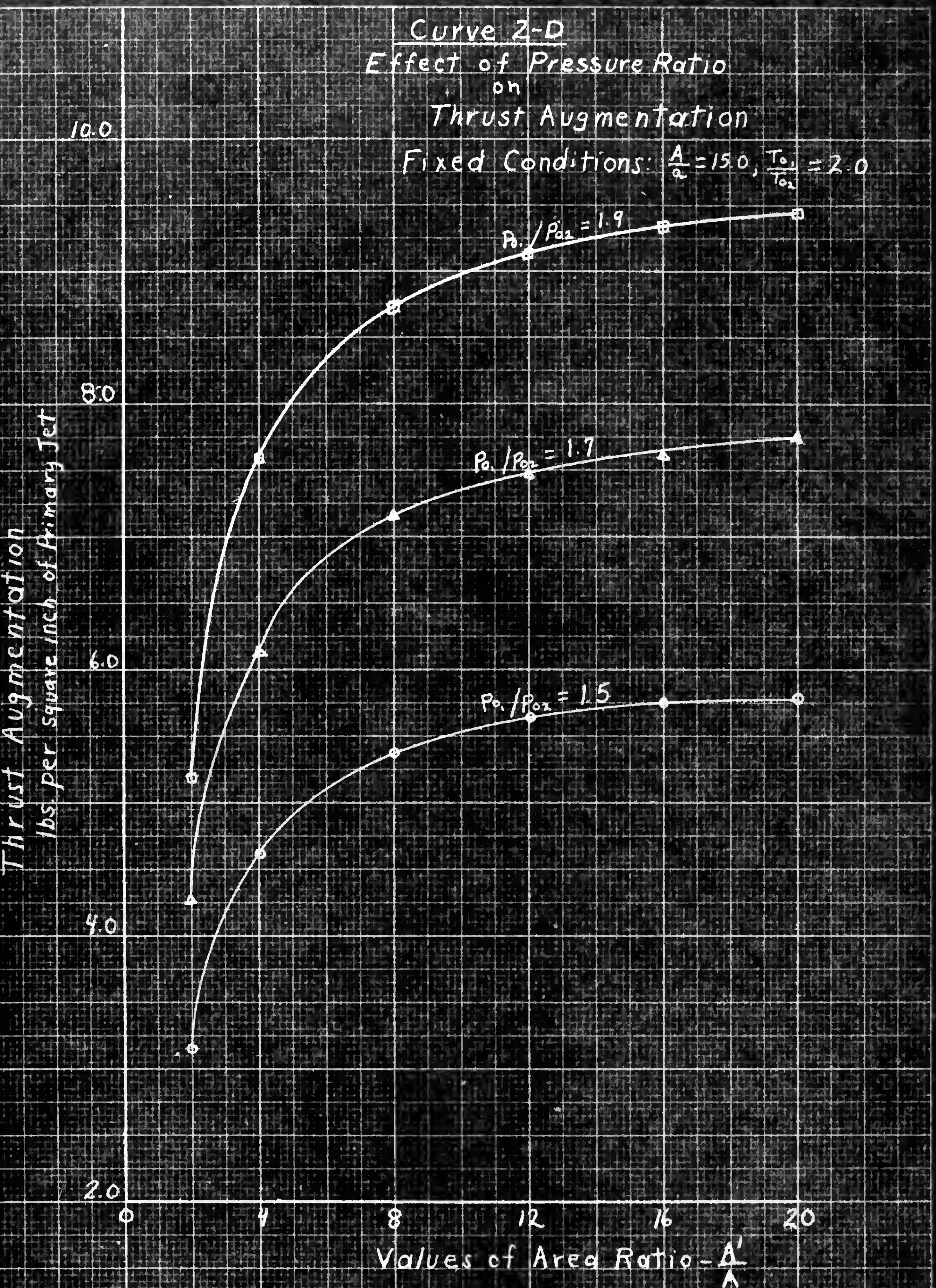
Effect of A/a on Thrust Augmentation

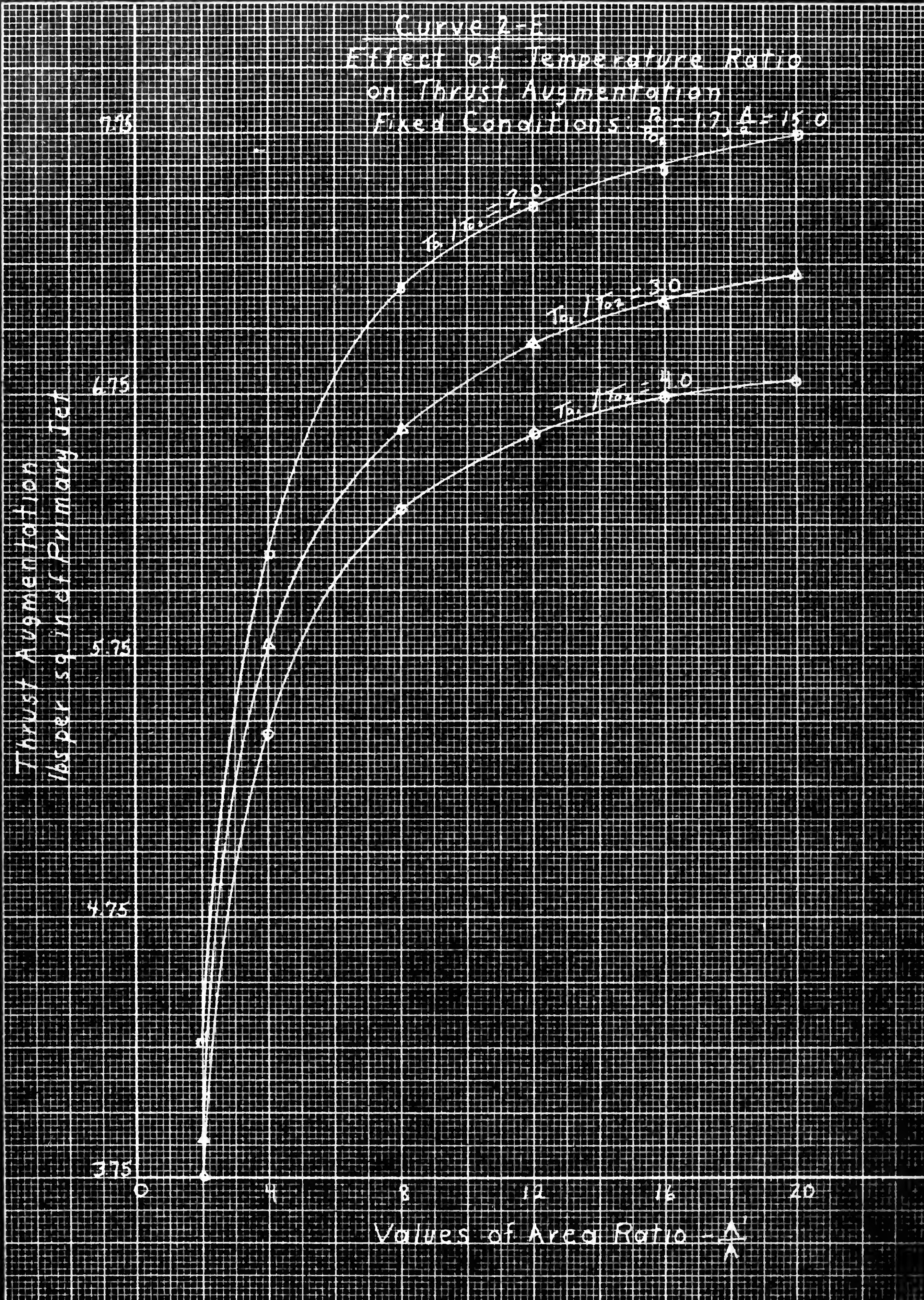
A/a	% of $A/a = 20$
5	49.2
10	73.3
12.5	83.2
15.0	89.0
17.5	94.7
20.0	

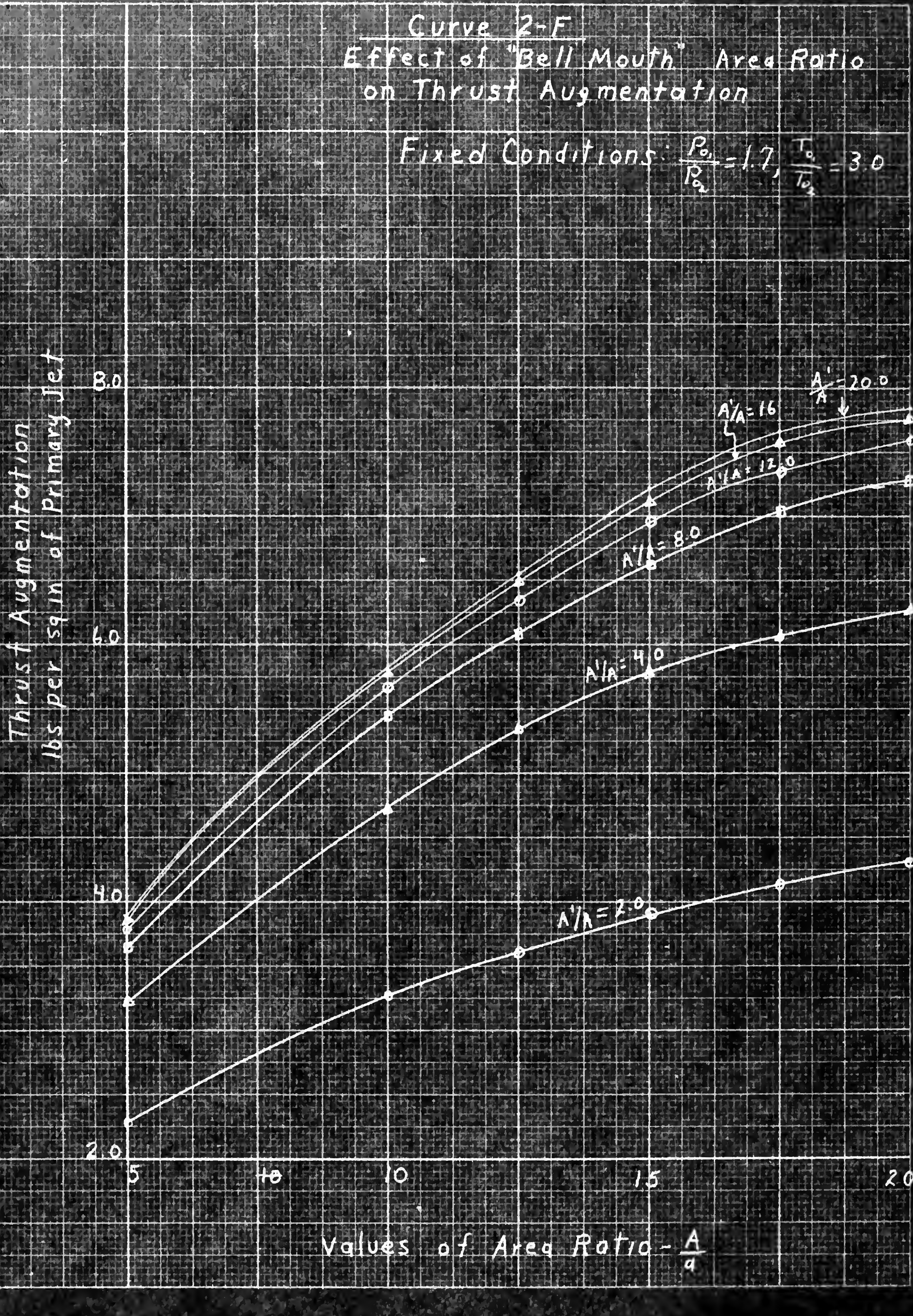
$P_{o_1}/P_{o_2} = 1.9$

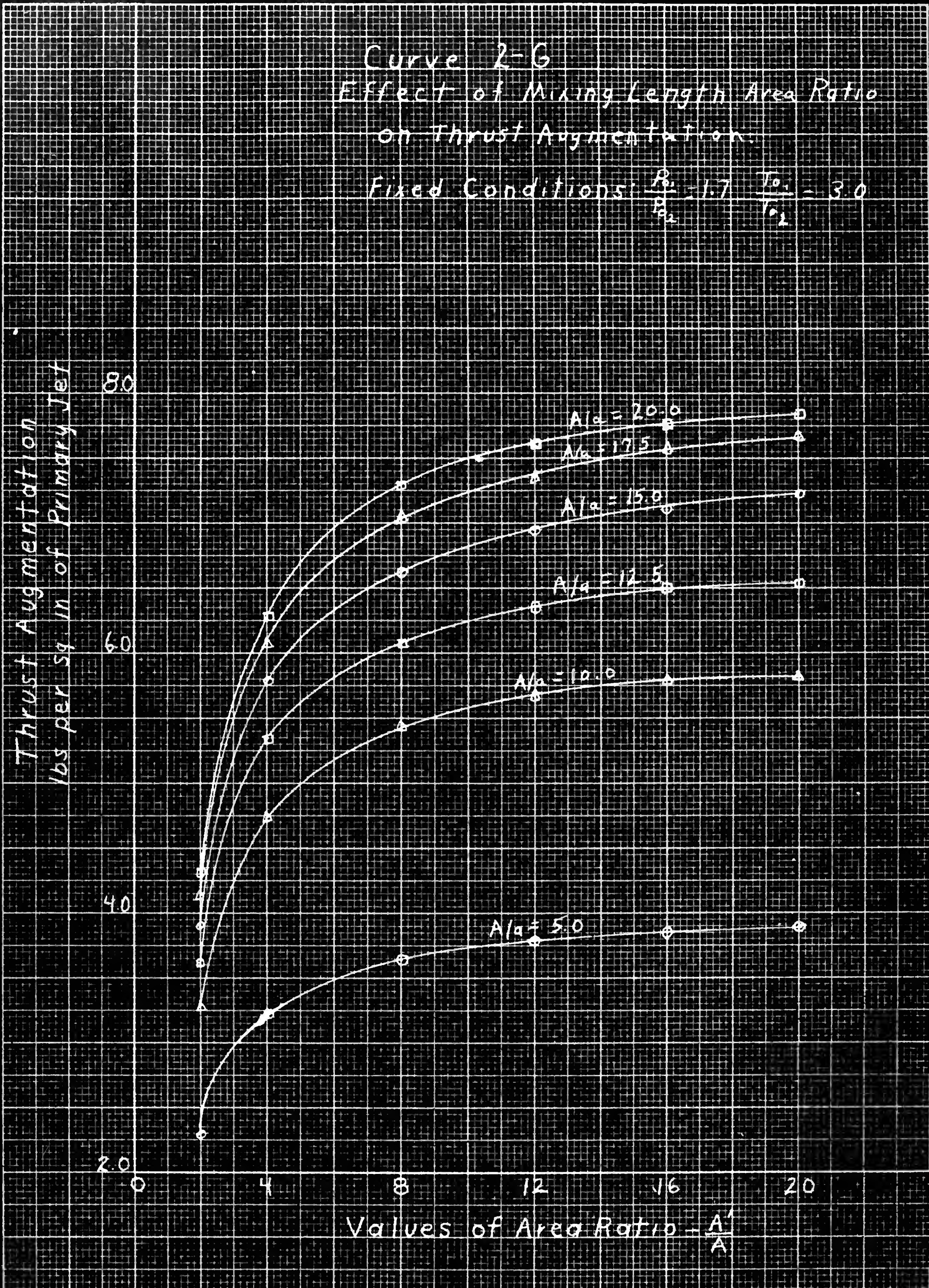
$T_{o_1}/T_{o_2} = 3.0$

$A'/A = 12$


Curve 2-D
Effect of Pressure Ratio
on
Thrust Augmentation
Fixed Conditions: A/a = 15.0, To1/To2 = 2.0
Po1/Po2 = 1.9
Po1/Po2 = 1.7
Po1/Po2 = 1.5
Thrust Augmentation
lbs. per square inch of Primary Jet
10.0
8.0
6.0
4.0
2.0
0
4
8
12
16
20
Values of Area Ratio - A'/A


Curve 2-E
Effect of Temperature Ratio
on Thrust Augmentation
Fixed Conditions $\frac{P_0}{P_a} = 1.7$, $\frac{A}{a} = 15.0$
$T_{0_1}/T_{0_2} = 2.0$
$T_{0_1}/T_{0_2} = 3.0$
$T_{0_1}/T_{0_2} = 4.0$
7.75
6.75
5.75
4.75
3.75
0
4
8
12
16
20
Thrust Augmentation
lbs per sq in of Primary Jet
Values of Area Ratio - $\frac{A'}{A}$


Curve 2-F
Effect of "Bell Mouth" Area Ratio on Thrust Augmentation
Fixed Conditions: $\frac{P_{o_1}}{P_{o_2}} = 1.7$, $\frac{T_{o_1}}{T_{o_2}} = 3.0$
Thrust Augmentation lbs per sq in of Primary Jet
Values of Area Ratio - $\frac{A}{a}$
$\frac{A'}{A} = 20.0$
A'/A = 16
A'/A = 12.0
A'/A = 8.0
A'/A = 4.0
A'/A = 2.0
8.0
6.0
4.0
2.0
5
10
15
20



Curve 2-G
Effect of Mixing Length Area Ratio on Thrust Augmentation.
Fixed Conditions: $\frac{P_{o_1}}{P_{o_2}} = 1.7$ $\frac{T_{o_1}}{T_{o_2}} = 3.0$
Thrust Augmentation
lbs per sq in of Primary Jet
8.0
6.0
4.0
2.0
A/a = 20.0
A/a = 17.5
A/a = 15.0
A/a = 12.5
A/a = 10.0
A/a = 5.0
0
4
8
12
16
20
Values of Area Ratio - $\frac{A'}{A}$

CONCLUSIONS

In designing a thrust augmentor the initial conditions of pressure ratio and temperature ratio would probably be fixed within narrow limits. This would set the conditions of two of the variables. However, if these two are permitted to be varied it appears that it would be wise to pick the highest pressure ratio available with the lowest temperature ratio. Since this is an anomaly the percentage figures indicate that of the two pressure ratio is far more important since a .2 change in pressure ratio increases thrust augmentation by approximately 20% but that a change in temperature ratio of 1 means only a 3 or 4% change in augmentation. It appears then, that temperature ratio as a variable is of relatively minor importance.

With temperature ratio and pressure ratio fixed, the other two variables are concerned with the physical limitations of the augmentor. Since in most cases weight and size limitations would make desirable a small augmentor it would be best to choose as small an area ratio as is practicable with performance characteristics. From the percentage calculations it is seen that an increase in the area of the bell mouth does not give a proportunate increase in thrust as it is increased above A'/A of 8. For a two and a half times increase in A'/A, the thrust increases only about 8%. As the size of the ratio approaches 20, the percentage increase in thrust augmentation is very

small. It can be concluded that a value of A'/A of from 8 to 10 is most practical. It is interesting to note that an A'/A of only two (which would mean a radius ratio increase of only 1.4) gives more than 50% of the thrust of A'/A equal 20.

Changes in A/a have a greater effect on augmentation. Increasing the ratio from 5 to 20 gives 50% more thrust. An A/a of 15 gives about 90% of the thrust obtainable from A/a of 20. It can be concluded that an A/a of 20 is probably the best but if space is limited an A/a of 15 will lose only 10% of the thrust.

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